Options for a non-dedicated test of the Pioneer anomaly

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Abstract

The Doppler-tracking data of the Pioneer 10 and 11 spacecraft show an unmodelled constant acceleration in the direction of the inner Solar System. Serious efforts have been undertaken to find a conventional explanation for this effect, all without success at the time we are writing. Hence the effect, commonly dubbed the Pioneer anomaly, is attracting considerable attention. We discuss strategies for an experimental verification of the anomaly via a space mission. Emphasis is put on two most plausible scenarios, non-dedicated concepts employing either a planetary exploration mission to the outer Solar System or a piggy-backed micro satellite to be launched from a mother-spacecraft travelling to Saturn or Jupiter. The study analyses the impact of a Pioneer anomaly test on the system and trajectory design for these two paradigms. It is found that both paradigms are capable of verifying and characterising the Pioneer anomaly without hampering the planetary exploration goals of the missions by a suitable adaption of the system design and introducing some minor mission analysis modifications.

Nomenclature

$A_{S/C}$	cross section area of spacecraft
$a_\odot$	acceleration by Solar radiation pressure
a_H	Hubble acceleration
c	speed of light
$\mathbf{e}_A$	unit normal vector of area A
F	force
f	one of the tracking observables s, v, α
g_0	gravitational acceleration at the Earth's surface
I	moment of inertia
I_{sp}	specific impulse
k	Boltzmann constant
M_α	total mass of α-particles produced by radioactive decay
$M_{S/C}$	spacecraft wet mass

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H_0	Hubble constant
P_a	asymmetrically radiated power
P_{tot}	total radiated power
$P_{\odot}$	Solar radiation constant
p_{tank}	pressure in tank
p_s	stagnation pressure
R	universal gas constant
r	heliocentric distance
$r_{\oplus}$	mean radius of Earth orbit
r_{pP}	radius of pericenter of swing-by
s	geocentric distance of spacecraft
s^*	deviation from nominal spacecraft trajectory
T	temperature
T_{tank}	temperature of fuel in tank
T_s	stagnation temperature
T_0	nominal temperature
t	time
t_e	time of departure at Earth
t_P	time of arrival/swing-by at planet
$t_{\oplus}$	orbital period of Earth
V_P	heliocentric velocity of planet
v_{in}	inbound asymptotic velocity
v_{out}	outbound asymptotic velocity
v_{α}	velocity of α-particles
$v_{\oplus}$	mean heliocentric velocity of the Earth
$\alpha_{\oplus}$	longitude in geocentric ecliptic coordinate system
α^*	deviation from nominal geocentric azimuth angle
β	angle between Earth–spacecraft direction and direction of anomaly
$\beta_{\odot}$	angle between Sun–spacecraft direction and direction of anomaly
$\beta_{\oplus}$	Earth–spacecraft–Sun angle
γ	flight angle
Δa	systematic uncertainty of acceleration a
Δf	uncertainty of tracking observable f
Δs	systematic uncertainty of geocentric distance s
Δv	systematic uncertainty of velocity v
ΔM	mass of expelled propellant
ΔV	velocity increment
$\Delta\epsilon$	change of emissivity per angle
$\Delta\epsilon_{max}$	maximal change of emissivity
$\Delta\mu_{\odot}$	change of the effective reduced Solar mass
ϵ_0	nominal emissivity per angle
η	specular reflectivity
θ	angle enclosed by $\mathbf{e}_A$ and $\mathbf{e}_{\odot}$
κ	adiabatic exponent
μ_P	reduced mass of planet

$\mu_\odot$	reduced Solar mass
ρ	true heliocentric distance
ψ	azimuth angle of cylinder coordinates
ϕ	mean anomaly of Earth orbit
σ	standard deviation
ω	rotational velocity of spacecraft

Superscripts

*	anomalous

Subscripts

$_{\text{track}}$	tracking error
$_0$	at time $t = 0$, i. e. beginning of measurement
$_\parallel$	parallel to Earth–spacecraft vector
$_\perp$	orthogonal to Earth–spacecraft vector

Introduction

In April 2004 the European Space Agency (ESA) invited the scientific community to participate in a Call for Themes for Cosmic Vision 2015-2025 to assist in developing the future plans of the Cosmic Vision programme of the ESA Directorate of Science. Amongst the 32 proposals received in the field of Fundamental Physics five were proposing a space experiment to investigate the so called Pioneer anomaly, i. e. an anomalous acceleration measured on the Pioneer 10 and 11 trajectories. In its recommendation for the Cosmic Vision programme, the Fundamental Physics Advisory Group (FPAG) of ESA considered these proposals as interesting for further investigations[1]. In view of the controversial discussion still surrounding the effect, and its high potential relevance for our understanding of the laws of physics, the FPAG recommended to ESA to study the possibility to investigate the putative anomaly on-board a non-dedicated exploration mission.

Motivated by this important discussion the paper wants to be a preliminary assessment of the capabilities of a mission to the outer Solar System for an investigation of the Pioneer anomaly. We identify two classes of missions that could well represent a future exploration mission. The first class is that of low-mass low-thrust orbiter missions to the outer planets. The second class is that of a heavy nuclear-reactor powered spacecraft, as currently being developed by NASA's Prometheus Program, to explore the giant planets. Within these two paradigms we analyse missions to all planets from Jupiter outward and consider to what extent a verification and characterisation of the Pioneer anomaly is possible. We will not discuss here what the scientific reasons might be to go to these planets (it would though be easy to find enough arguments to support any mission to any of these distant objects) as we would rather embrace the simpler perspective that has recently be taken in an article of the Economist[2] observing that "all scientific disciplines begin with stamp collecting (in other words gathering examples without really knowing what to do with them). They then progress to classification (when there are enough samples for patterns to emerge). After that, with a bit of luck, comes understanding." The data obtained from past missions to outer Solar System planets (namely the Voyagers), are no longer sufficient to satisfy the needs of the scientific community for planetary data and new missions are certainly needed that can provide greater details than simple fly-by missions. This is why there is no doubt that orbiter missions to the outer planets and beyond will be designed and launched in the not-so-far future.

The layout of our considerations is the following: we begin with a review of the Pioneer anomaly. Particular emphasis is put on the significance of the data from the Pioneer spacecraft and on the considerations that have been put forward for their explanation. The theoretical models proposed to explain the anomaly in terms of new physics are reviewed with particular emphasis on the demands that the various patterns of explanation introduce on the design of an upcoming experiment. We formulate the minimal experimental requirements on the spacecraft and on the trajectory of a test mission. We then briefly review existing dedicated mission concepts in order to see what lessons can be drawn from them and applied to a non-dedicated mission. These preparatory considerations are followed by system design and mission analysis: we describe the two paradigms to be considered and give a preliminary mass and power budget that forms the basis of our analysis. We assess how the requirement of low-acceleration systematics can be met. Consequently, radio-tracking of the spacecraft for the two paradigms is discussed and its performance for the detection and characterisation of the anomaly is evaluated. The tracking performance is found to depend on the trajectory parameters and the criteria for a suitable orbit are derived. After that we we briefly touch upon the possibility to use a special payload to support the Pioneer anomaly test. Based on the infered orbit criteria and our knowledge of the Pioneer anomaly the space of trajectory options is explored. The Pareto fronts of a multi-objective optimisation between the main mission goals and the objective to investigate the Pioneer anomaly are determined. The major conclusions for the spacecraft and the trajectory design as well as the performance of the missions are summarised in the final section.

The Pioneer anomaly

In 1974 the Pioneer 10 orbit changed to a hyperbolic escape trajectory by a Jupiter gravity assist. The ascending node of the asymptote was (and since remained) $-3.4\,\mathrm{deg}$, the inclination of the orbit is $26.2\,\mathrm{deg}$. In 1980 Pioneer 11 conducted its Saturn swing-by and went on a hyperbolic trajectory with an asymptotic ascending node of $35.6\,\mathrm{deg}$ and an inclination of $9.5\,\mathrm{deg}$. The orbit determination for both crafts relied entirely on Doppler tracking. Already before the Jupiter swing-by the orbit reconstruction for Pioneer 10 indicated and unmodelled deceleration of the order of $10^{-9}\,\mathrm{m/s^2}$ as first reported by Null[3]. This effect was, at that time, attributed to on-board generated systematics, in particular to fuel leaks. However, also on the hyperbolic coast, an unmodelled deceleration remained, although the number of attitude control manoeuvres was reduced to approximately one every 5 months. Hence fuel leakage, triggered by thruster activity, could no longer be considered as an explanation. Even more surprising also the Doppler tracking of Pioneer 11 show an unmodelled deceleration of a similar magnitude. The anomaly on both probes has been subject to three independent analysis that used different orbit determination programs[4,5,6]. The result of all these investigations was that an anomalous Doppler blue shift is present in the tracking data of both craft and that the magnitude of the blue shift is approximately 1.1×10^{-8} Hz corresponding to an apparent deceleration of the spacecraft of approximately $8 \times 10^{-10}\,\mathrm{m/s^2}$. It is worth emphasising that from the Doppler data alone it is not possible to distinguish between an anomalous frequency shift of the radio signal — in conventional terms this could also indicate a drift of the Deep Space Network clocks — and a real deceleration of the spacecraft. The observational data and the following analysis are described in detail in the work of Anderson et al.[5] and Markwardt[6]. The results of these different analyses show a discrepancy at a level of approximately 5% of the inferred deceleration. Unfortunately, none of the analysis performed make use of the entire data set available.

The quality of the data is best judged from the plot of the Pioneer 10 anomalous acceleration as determined by the CHASMP software (developed by the Aerospace Corporation) and reported by Anderson[5]. While it is quite obvious that the data show the existence of an anomalous acceleration it is also obvious that the variation of the measured anomaly, due to systematics, is too big to evaluate the first derivative of the anomaly.

Systematics?

Many attempts[7,8,9,10,11,12,13,14,15] have been made to interpret the anomaly as an effect of on-board systematics ranging from fuel leakage to heat radiating from the spacecraft. In the work of Anderson et al.[5] it is however concluded that none of the effects considered is likely to have caused the anomaly. They argument that a heat generated anomaly would be mainly due to the RTG's heat and that this can be excluded because the heat decay with the Plutonium half-life of 87.7 years, would have shown up as a decrease of the deceleration in the longest analysed data interval for Pioneer 10, ranging from January 1987 to July 1998. They proceed noting that gas leaks can also be excluded as the cause of the anomalous deceleration under the sole assumption that the amount of fuel leakage is uncorrelated between the two craft. Hence also gas leaks seem to be an unlikely reason for the anomaly in view of the fact that the same leak would have occurred on Pioneer 10 and 11. However, since both spacecraft have an identical design, this origin of the anomaly can ultimately not be excluded. Unfortunately, the conclusions of the various quoted studies are far from unanimous. At the current stage of investigation it is not clear if one should attribute the anomaly to a conventional effect or consider explanations routed in new physical phenomena. A complete examination of the full archive of Doppler data is certainly needed. Nevertheless, even with this enhanced knowledge it seems highly doubtful that the issue can be decided, since there exist considerable uncertainties in the modelling of forces generated on-board Pioneer 10 and 11. In view of the necessity of an improved evaluation of the Doppler data the authors feel obliged to express their unease about the discrepancy between the results obtained with the different orbit determination programs. In particular it is noteworthy that the disagreement between the three analysis is bigger that their nominal errors.

New physics?

Although the Pioneer anomaly is an effect at the border of what is detectable with radiometric tracking of a deep-space probe, it is huge in physical terms. The anomaly exceeds by five orders of magnitude the corrections to Newtonian motion predicted by general relativity (at 50 AU Solar distance). Hence, if the effect was not due to systematics, it would have a considerable impact our models of fundamental forces, regardless if the anomaly was due to a deceleration of the spacecraft or a blue shift of the radio signal. One of the obstacles for attempting an explanation of the Pioneer anomaly in terms of new physics is that a modification of gravitation, large enough to explain the Pioneer anomaly, is in obvious contradiction to the planetary ephemerides. This becomes particularly clear if one considers the orbit of Neptune. At 30 AU the Pioneer anomaly is visible in the Doppler data of both Pioneer 10 and 11. The influence of an additional radial acceleration of $8 \times 10^{-10}\mathrm{m/s^2}$ on Neptune is conveniently parameterised in a change of effective reduced Solar mass, $\mu_\odot$ felt by the planet[16]. The resulting value, $\Delta\mu_\odot = 1.4 \times 10^{-4}\,\mu_\odot$, is nearly two orders of magnitude beyond the current observational constraint of $\Delta\mu_\odot = -1.9 \pm 1.8 \times 10^{-6}\,\mu_\odot$. Similarly Pioneer 11 data contradict the Uranus ephemerides by more than one order of magnitude. Thus, the Pioneer anomaly can hardly be ascribed to a gravitational force as this would indicate a considerable violation of the weak equivalence principle. In particular, planetary constraints rule out an explanation in terms of a long-range Yukawa force[5,17]. Other, more subtle, explanations are to be attempted. Already in the first paper discussing the Pioneer anomaly it was noted that the magnitude of the effect coincides with the Hubble acceleration and with the so-called MOND parameter[4]. Subsequently there have been several attempts to associate the Pioneer anomaly both to the cosmic expansion and to the MOND model.

The Hubble acceleration is formed by converting the Hubble expansion rate[18], $H_0 = (71 + 4 - 3)\,(\mathrm{km/s})/\mathrm{Mpc}$, to an acceleration by multiplying it by the speed of light, $c = 3 \times 10^8\mathrm{m/s}$, (which is the only fundamental constant, that can do this conversion)*, $a_H \equiv cH_0 = (6.9 \pm 0.7) \times 10^{10}\mathrm{m/s^2}$. Attempts to connect the Pioneer anomaly with the cosmic expansion consider both possibilities, that the Pioneer anomaly affects only the light propagation[19,20] or that it causes a real deceleration of the spacecraft[21,22].

However the predominant opinion, starting with the work of Einstein and Straus[23], is that the cosmic dynamics has far too little influence to be visible in any physical processes in the Solar System. The case has recently been reviewed confirming the common opinion[24]. Other problems of this approach are the apparent violation of the weak equivalence principle associated with the Pioneer anomaly and the opposite signs of the cosmic expansion and of the Pioneer anomaly.

Modified Newtonian Dynamics (MOND) is a long distance modification of Newtonian gravity which successfully explains the dynamics on galactic scales without invoking dark matter[25], (see Sanders et al.[26] for a review). The MOND parameter, $(1.2 \pm .3) \times 10^{-10}\mathrm{m/s^2}$, gives the acceleration scale at which the gravitational force changes from the Newtonian law to the MOND law, that predicts stronger gravitational attraction. While MOND is consistent and successful as a non-relativistic theory its relativistic generalisations remain unsatisfactory because they require a fixed

*The Hubble acceleration is by no means an artificial construct but is related to actual observables. For instance it describes the lowest order correction from the cosmic expansion to the length of light-rays from a past event to a present-day observer $d = c\,\Delta t + \frac{a_H}{2}(\Delta t)^2$.

background structure or even have acausal features[27]. The Pioneer anomaly can be connected with MOND if one assumes that the transition between the Newtonian and MOND regimes can be approximated by a Taylor series around the Newtonian potential and that the MOND parameter sets the magnitude of the first term in this Taylor expansion[27,28].

In order to circumvent the constraints from planetary ephemerides, momentum-dependent "nonlocal" modifications of general relativity have also been considered[29,30]. Whereas the original idea is rather vague, a more elaborate model[30] faces several severe problems. Jaekel and Reynaud[17] introduced two different momentum-dependent gravitational constants for the trace and the conformal sector of the Einstein equations. Such running couplings lead to a violation of the Bianchi identities unless one resorts to a non-local reformulation of the Einstein-Hilbert action[31,32]. Even then causality of the resulting physical laws needs careful consideration. Even worse, this modification results in an unstable dipole-ghost (cf. Smilga[33]). It seems hard to conceive that the combination of instability and fine-tuning between the scalar and conformal sectors can result in a viable model.

There are several other works pursuing even more unusual lines of explanation. The reader is advised to refer to the papers by Anderson et al.[5] and by Bertolami and Paramos[34] for reviews of some of the explanations the Pioneer anomaly relying on more exotic physics.

Other spacecraft

It stands to reason that, if the anomaly detected in the tracking data of the Pioneers was due to some unknown fundamental physical phenomena, the same should be observed in the data from other mission. This issue has already been analysed for the Voyager spacecraft and for Galileo and Ulysses[5]. The basic results are that the 3-axis stabilisation of the Voyager probes performs so many attitude-control manoeuvres that it is impossible to detect an anomalous acceleration of the magnitude under consideration on these spacecraft. The case is similar for Galileo and Ulysses where the large systematics due to Solar radiation pressure and malfunctions of part of the attitude control systems prohibited any reliable result. Also the Cassini tracking did not yield results of the desired precision as the spacecraft is 3-axis stabilised and the mounting of the RTG's causes a large acceleration bias[35].

At first sight ESA's Rosetta mission[36] to the comet Churymov-Gerasimenko seems to be a good upcoming candidate to verify the Pioneer anomaly. The Rosetta trajectory has a long elliptic coast arc from July 2011 to January 2014, during which the distance from the Sun will be between 4.5 and 5.4 AU. During the coast arc the Rosetta craft will enter a so-called hibernation mode when the power generated by the Solar array drops below a certain value. In this mode the spacecraft will be spin-stabilised with a rotational velocity of approximately 1 rpm. Most on-board instruments, including the attitude control and radio transmission system, will be switched-off. Unfortunately during the hibernation no tracking can be performed, hence the presence of a force can only be inferred from the trajectory evolution between the entry and exit of hibernation. The large $68\,\mathrm{m}^2$ Solar arrays on the craft will cause, also during this phase, an acceleration bias of approximately $10^{-8}\,\mathrm{m/s}^2$, one order of magnitude larger than the Pioneer anomaly. As the orientation of the Solar array during the hibernation phase is not actively maintained a large uncertainty in the Solar radiation force on the spacecraft will result. Hence a test of the Pioneer anomaly with Rosetta is not possible.

A mission, which is closer to the class of exploration missions discussed in this work, is NASA's New Horizons mission[37]. The destination of this mission is Pluto and the launch is scheduled for 2006. During most of the journey the spacecraft will be in a spin-stabilised mode with little on-board activity, similar to that of Rosetta. In contrast to Rosetta this mode is not required by power constraints and was mainly chosen to increase component lifetime and reduce operation costs. Hence an enhanced tracking of the mission for a test of the Pioneer anomaly would be possible. Currently no such activity is foreseen. Unfortunately, the system design of the mission is far from ideal for a test of the Pioneer anomaly because the RTG is directly attached to the spacecraft bus. This design will lead to a considerable backscattering of RTG heat from the back of the antenna causing an acceleration bias along the spin axis of the spacecraft. Without a purpose-made high-accuracy thermal model and an active monitoring of possible degradation of the surface properties of the RTG and the back of the antenna this effect will be very difficult to discriminate from a putative anomaly (see below for a discussion of this problem for the general case). Hence a test of the Pioneer anomaly with the New Horizons spacecraft would only lead to unreliable results.

Existing dedicated mission concepts

Before we turn to a discussion of the possibilities to test the Pioneer anomaly on a non-dedicated mission we review the three existing suggestions for dedicated missions that may be found in the literature.

Symmetrised Pioneer

This concept, proposed by Nieto and Turyshev[35] aims, broadly speaking, at a duplication of the Pioneer design with reduced spacecraft inherent systematics. In this concept the spin stabilisation of the craft, the use of RTG's on booms as power sources, and the use of the radio tracking to detect the anomaly, stem from the design of the Pioneer probes. The measurements shall take place on a hyperbolic escape trajectory. Also the mass, moment of inertia and rotational velocity of the craft are envisaged to lie in the magnitude of the Pioneer 10 and 11 values. The major improvement of the design consist of a fore–aft symmetrical design of the spacecraft including in particular two permanently operating radio transmission antennas. This design would lead to an approximately symmetric radiation and leakage characteristics of the spacecraft. A possible acceleration bias would be detected by turning the spacecraft around several times during the measurement phase. The measurement precision of this concept has been estimated to $6\times10^{-12}\,\mathrm{m/s^2}$. We will comment on this values after discussing the various systematic sources of acceleration in detail.

Another suggestion, in which the measurement is also conducted by radio tracking, has been put forward by Bertolami and Tajmar[38]. Their mission is based on the use of a small, highly symmetric, e. g. spherical, spacecraft on an elliptical orbit with an aphelion of 3 to 5 AU.

Mother–Child formation

A totally different approach to the verification of the Pioneer anomaly is taken by Chui and Penanen[39]. They consider a two spacecraft system consisting of a mother-ship and a small passive satellite with retro reflectors attached. This child craft would be similar in design to the Lageos 2 satellite. The mother-ship is supposed to follow the free coast of the passive satellite in a distance of the order of km. The trajectory of the passive satellite will be analysed for any sign of the Pioneer anomaly. The tracking is envisaged to be accomplished by laser ranging of the passive satellite from the mother-craft and laser ranging of the mother-craft from Earth or Earth orbit. Both distances shall be determined by a pulsed laser. While the signal will just be reflected at the passive satellite, the laser signal from Earth needs to be transponded at the mother-ship due to the large distance between Earth and the mother-ship. The envisaged concept is claimed to be capable of extremely high accuracy, $10^{-14}\,\mathrm{m/s^2}$ according to Penanen and Chui[39], and hence promises additional scientific return by being able to detect the gravitational force exerted by the Galaxy. The price to pay for the high accuracy is a complex system design requiring a powerful laser ($P \sim 10\,\mathrm{W}$) on-board the mother-craft for Earth communications and eventually even a receiver station in Earth orbit to overcome attenuation by the atmosphere. Unfortunately, several doubts arise concerning the feasibility of the method and its actual suitability for a test of the Pioneer anomaly. An experiment to investigate the Pioneer anomaly should be, as explained above, sensitive to both a force on the spacecraft and a phase-shift of the transmission signal. Since the method envisaged by Penanen and Chui uses the travel time of the laser pulse and simple pulse counting for the satellite tracking, the tracking system is not sensitive to phase-shifts of the laser signal. Hence one of the candidate sources of the Pioneer anomaly cannot be investigated with the proposed mission.

A modified version of this concept is currently favoured in the "Consolidated proposal for a mission to test the Pioneer Anomaly" as proposed to ESA in response to the Call for Themes "Cosmic Vision 2015–2025"[40].[†] In this proposal the tracking over the Earth–mother-craft distance is done via radio communications. This enhances the concept for the possibility to detect a blue shift in the radio signal. The short distance between the mother-craft and the child would not influence the blue shift as it would be negligible for a cumulative effect over the travel distance. This use of radio waves for the long distance tracking also considerably relieves the technology development requirements for the mission as the laser for the short distance tracking can be of considerably lower power than that for long distance tracking to Earth.

Nevertheless considerable technology developments would be necessary: since the experiment will have to be conducted in the outer part of the Solar System a mission duration of a decade or more seems realistic. However currently no lasers are available which have a sufficient lifetime. The results of the NASA's Lidar In-space Technology Experiment (LITE) on-board the space shuttle showed a considerable degradation of pulse energy which demonstrates the difficulties of developing laser with sufficient lifetimes[41].

In its modified version the mother-child mission concept seems well suited for a test of the Pioneer anomaly. The concept is however considerably more complex that the symmetric spacecraft option

[†]This concept originated from a suggestion by U. Johann and R. Förstner independently of the work of Chui and Penanen[39].

and a reliable estimate for the expected precision of the experiment is still lacking.

Non-dedicated mission concepts

The capabilities of exploration missions

Exploration missions to the outer Solar System will naturally take place with the continued efforts to gain new insights into the formation of the Solar System. Such missions will naturally offer an opportunity to test the Pioneer anomaly. Missions to Uranus, Neptune or Pluto will most naturally feature a Jupiter gravity assist followed by a hyperbolic coast arc. This coast phase lends itself to a precision tracking of the spacecraft trajectory which can be analysed to detect anomalous accelerations. The major design drivers for such a mission would however be the planetary exploration goals. Hence a design like the symmetric spacecraft described above would be prohibited by payload requirements and the need to accommodate a propulsion module capable of achieving a capture into the orbit of an outer planet. Also a special experimental payload for a Pioneer anomaly test will most probably be excluded by mass constraints. However even under these conditions a verification of the Pioneer anomaly is still attainable. Although additional requirements on the spacecraft design are imposed these requirements can be fulfilled at no additional mass, little to no impact on the other observational program of the satellite and no additional risks. To be specific we will consider a class of low-mass, low thrust missions derived from the study of a Pluto orbiter probe, POP, and demonstrate the feasibility of a Pioneer anomaly test on such a mission[42,43].

Another interesting class of future missions that may allow a Pioneer anomaly test are large spacecraft with electric propulsion powered by a nuclear reactor to explore the moons of the giant planets Jupiter and Saturn. One such spacecraft is currently considered by NASA under the name of Jupiter Icy Moons Orbiter, JIMO. While the large amount of heat radiated from the nuclear reactor on the craft would prohibit a test of the Pioneer anomaly on the main spacecraft, this class of missions may accommodate a small daughter spacecraft of less than a 200 kg mass (This is to be compared to the 1500 kg of payload envisaged for JIMO.). This spacecraft could then be jettisoned during the approach of the mother-craft to target planet, and use this planet for a powered gravity assist to go on a ballistic hyperbolic trajectory. The Pioneer anomaly test would then be performed by the daughter craft. In order to have an idea on the spacecraft relevant data we now give a preliminary mass-budget for the the two different concepts described above.

The POP spacecraft

Pluto Orbiter Probe (POP) is an advanced spacecraft designed within the Advanced Concepts Team of ESA[42,43,44,45] able to put a 20 kg payload onto a low altitude Pluto orbit. The preliminary design has a dry mass of 516 kg and a wet mass of 837 kg. The spacecraft is powered by four RTG's. The original mission profile envisages a launch in 2016 and arrival at Pluto after 18 years of travel time and a Jupiter gravity assist in 2018. A suitable launch vehicle would be an Ariane 5 Initiative 2010. The preliminary design of POP consists of a cylindrical main structure, of 1.85 m length and 1.2 m diameter. On one end of the main structure the 2.5 m diameter Ka-band antenna is mounted. The four GPHS RTG's are placed on the other end of the main structure inclined 45 deg to the symmetry axis of the craft. The 4 QinetiQ T5 main engines are as well placed at this end of the main structure. Next to the main engines in the main structure is the propellant tank accommodating 270 kg of Xenon propellant. POP is a good example of what an advanced spacecraft travelling to the outer Solar System may look like and we therefore take it as a paradigm of these kind of missions. In table 1 the key figures of this paradigm, that are relevant for our considerations, are given.

The piggy-back spacecraft

In the framework of NASA's Prometheus Program JIMO has been proposed by NASA as the first mission to demonstrate the capabilities of electric propulsion powered by a nuclear reactor. A preliminary design of the mission considers an 18000 kg spacecraft equipped with high specific impulse engines capable of 2 N of thrust. An heavy launcher would put the spacecraft into a high altitude Earth orbit from where the spacecraft would start to spiral out until a Moon swing-by would put the spacecraft into a heliocentric trajectory to Jupiter. The final goal is to be captured by Jupiter and to explore three of its largest Moons, namely Ganymede, Callisto and Io. For JIMO a science payload of 1500 kg is envisaged, 25% of which are expected to be made up by a Europa lander [46].

In view of the large scientific potential of JIMO it is likely that the mission will be followed by similar ones. Due to its high payload capabilities the JIMO mission and its successors could also carry a small spacecraft to carry out a Pioneer anomaly test. The spacecraft would then be jettisoned at some point of the trajectory and put into an hyperbolic heliocentric trajectory via a planetary gravity assist. This would allow the spacecraft to perform a Pioneer anomaly test after its swing-by phase.

A possible baseline design for the piggy-back spacecraft, resulting form the design-driver to reduce on-board generated systematics (see below), could be a spin-stabilised craft. It would use ion thrusters (e. g. hollow cathode thrusters) for attitude-control, and carry only a minimal scientific payload. Since only a small data rate would be required a 1.5 m high-gain antenna would be sufficient even in the outer Solar System. The required 80 W of power to operate the payload, the communication subsystem and the AOCS would be provided by two RTG's weighting 12.5 kg each. Heat pipes from the RTG's to the main structure of the spacecraft would be used for thermal control. A preliminary mass estimate can be obtained based on the results of ESA's study of an Interstellar Heliopause Probe[47], which has a similar baseline. The result yields a mass of 150 kg. In addition a chemical propulsion module would be necessary to provide a moderate ΔV before and during the swing-by. This propulsion stage will be jettisoned after the swing-by to eliminate the danger that leakage of residual fuel from the module spoils the Pioneer anomaly test. The dry-mass of the module is estimated to be 16 kg. A detailed design is beyond the scope of this article. Hence we apply a 20% mass margin and a 20% margin on the required power. Systematic accelerations inherent to the spacecraft scale inversely to the mass of the spacecraft. Hence for the calculation of the error budget the conservative estimate will arise from assuming the lower mass for the spacecraft but the higher power consumption. The relevant parameters we considered for the piggy-back spacecraft are also summarised in table 1.

	POP	piggy-back
wet mass during coast / kg	750	150
electric power / W	1000	100
RTG heat / W	10000	1000
maximal radio-transmission power /W	50	10
antenna diameter /m	2.5	1.5

Table 1: Overview of relevant spacecraft data for the two mission paradigms.

Spacecraft design

We take the data presented on the two paradigms of future exploration missions and investigate the impact on the design of such spacecraft that result from having to perform a Pioneer anomaly test. We observe that the major obstacle to determine the nature of the anomaly present in the trajectory data from Pioneer 10 and 11 is not the accuracy of the tracking signal but a lack of knowledge concerning the systematic effects induced by conventional forces. Several effects generate forces, which lead to accelerations of the same order of magnitude as the observed anomaly at least during part of the data interval under consideration. The major uncertainties concerning the Pioneer probes were anisotropic heat emission and possible fuel leaks. Other important conventional forces for the mission profiles under consideration will be Solar radiation pressure, the thrust history of the spacecraft and the radiation force of the on-board-generated radio signal. The uncertainties in the forces generated by these effects need to be strongly reduced in order not to spoil a precision determination of a putative anomaly. For a mission, for which a test of the Pioneer anomaly is not the main objective, the possibilities to implement a reduction of these forces by adapting the spacecraft design will be limited. Nevertheless it is possible to considerably reduce the on-board generated forces and improve the ability to model them adopting from the early design phase some spacecraft design expedients, which do not spoil the planetary-science mission objectives.

Thrust history uncertainties

The knowledge of the thrust history of the spacecraft plays a major role for the ability to search for small forces acting on the spacecraft[48]. This is readily illustrated by the fact that the navigational precision of the Voyager spacecraft is two orders of magnitude below that of the Pioneer probes. This lower navigational accuracy is mainly due to the difference in the attitude control system of the two spacecraft models. Due to their 3-axis stabilisation the Voyagers have a high rate of control thruster firings[5] in order to maintain the nominal attitude of the spacecraft. However the thrust level of chemical or cold-gas control thrusters varies considerably from firing to firing. On top of this, the firing of a thruster usually is followed by a considerable “non-propulsive” outflow of propellant which generates accelerations easily exceeding the magnitude of the Pioneer anomalous acceleration (cf. Anderson et al.[5]). A more precise thrust history becomes available if ion engines are used for the control of the spacecraft. In addition electric-propulsion systems generate considerably smaller forces due to non-propulsive fuel outflow (see below).

The thrust history could be controlled very precisely if field effect emission thrusters (FEEP's) could be used for attitude control. These offer fine regulation of the thrust level and have practically no residual fuel outflow after the thruster is switched off. For the exploration scenario this choice is however prohibited by the thrust requirement imposed from the attitude control during the swing-by manoeuvre and in-orbit at the final destination. For the piggy-back paradigm FEEP's would generate sufficient thrust during the measurement coast but they would hardly suffice to provide attitude control during the powered swing-by. Hence FEEP's can only be employed if the propulsion module for the swing-by provides its own, more powerful attitude actuation system, which would add a further mass penalty to the mission.

A more efficient solution is to reduce the amount of attitude control manoeuvres. This is achieved by spin stabilisation of the satellite. For the piggy-back paradigm this poses no problems and it is convenient to choose a relatively high rotational velocity in order to guarantee the highest possible stability against disturbances. For the exploration mission spin stabilisation seems to be in contradiction to the requirements of planetary science as the instruments for the later required high pointing accuracy, pointing stability and slew rate capabilities are not provided by a spin stabilised spacecraft. In reality the requirements of a Pioneer-anomaly test and planetary science are not in contradiction as the different objectives have to be fulfilled in different parts of the mission. Hence the spacecraft can be in spin stabilised mode during the coast phase, which will be used for the search for new forces, and change to 3-axis stabilised mode when approaching its final destination. Also for eventual gravity assists 3-axis stabilised control is desirable as it allows for a more precise control of the nominal swing-by trajectory. The implementation of two stabilisation modes does not lead to profound complications.

The spin-up before and spin-down after the coast, in which the anomaly is tested, will be performed in deep space where little external disturbances act on the spacecraft. Hence the spin-up and spin-down can be conducted over a long time span and will only consume a negligible amount of propellant (see Izzo et al.[45]). Furthermore no additional attitude acquisition hardware will be necessary. Thanks to the low disturbance level in deep space the rotational velocity of the satellite can be very low, $\sim 0.01\,\mathrm{rpm}$, and the star trackers for the 3-axis stabilised mode would still be sufficient for attitude acquisition. Indeed the coast in spin stabilised mode might even save mass because it reduces the operating time of the momentum/reaction wheels or gyros and hence reduces the required level of redundancy.

One might also envisage that the spin of the spacecraft has an influence on the magnitude of the anomalous force (cf. [13,14,15] for an unsuccessful attempt, which tried to locate the origin of the anomaly in the rotation of the Pioneer probes.). Such a dependence may be reasonably excluded. The rotational speed of the Pioneer 10 spacecraft was 4.5 rpm to 4.2 rpm, the one of the Pioneer 11 was about 7.3 rpm to 7.2 rpm. Assuming a power-law dependence of the anomalous acceleration a on the rotational velocities of the crafts ω, $a = \mathrm{const}\,\omega^x$, the exponent is constrained by the error margin of the anomalous acceleration to $|x| < 0.7$. Thus, in particular, a linear dependence of the anomalous acceleration on the rotational velocity and a linear dependence of the anomalous acceleration on the rotational energy of the space-craft, $E_{\mathrm{rot}} = I\omega^2/2$ with I being the moment of inertia along the spin axis, is ruled out. Hence, a dependence of the anomaly on the rotational parameters of the spacecraft seems rather unlikely and in the following no requirements on the rotational velocity will be considered.

Fuel leaks and out-gassing

A fuel leak from the attitude control system presents one of the best candidates for a conventional explanation of the Pioneer anomaly. Unfortunately, also in a new mission, it would be difficult to entirely eliminate the possibility of fuel leaks caused by a malfunctioning valve. The force F generated by a mass flow rate $\dot{m}$ is given by (see Longuski and König[48]):

$$F = \dot{m}\sqrt{2RT_s\left(\frac{1+\kappa}{\kappa}\right)}\,.$$

For chemical propellant systems the stagnation pressure corresponds to the temperature in the tank $T_s = T_{\mathrm{tank}}$. Requiring that the maximal additional acceleration generated by propellant leakage should not exceed $10^{-11}\,\mathrm{m/s^2}$, then the maximally allowed forces are $F \lesssim 7.5 \times 10^{-9}\,\mathrm{N}$ in the planetary exploration scenario and $F \lesssim 1 \times 10^{-9}\,\mathrm{N}$ for the piggy-back spacecraft. The corresponding mass-flow rates allowed would therefore be less than 5 g/year assuming realistically tank temperatures higher than 100K. This requirement is far to demanding for a typical chemical attitude control system (cf. Longuski and König[48]).

The problem of fuel leakage becomes more manageable for electric propulsion systems, which do not rely on a high tank pressure to generate additional thrust. The propellant gas passes from the high-pressure tank at $\sim 150\,\mathrm{bar}$ and $\sim 300\,\mathrm{K}$ through a central pressure regulator before it is distributed to the engines at low pressure, $\sim 2\,\mathrm{bar}$. (Hence a redundant layout of the pressure regulator would reduce considerably the risk of leakage through a valve failure).

The internal leakage rate of a central pressure regulator in an electric engine piping is typically (without loss of generality we assume Xenon as a fuel) $\sim 10^{-8}\,\mathrm{lbar/s}$, and the external leakage is approximately $10^{-12}\,\mathrm{lbar/s}$. From these numbers it is immediate to work out that while external leakage is sufficiently under control for the purpose of a Pioneer anomaly test, it is desirable to further reduce internal leakage. This can be achieved by placing a small reservoir with a low-pressure valve after the central regulator. For the low pressure valve an even smaller internal leakage is attainable, while the reservoir accommodates the gas leaking through the regulator until the next thruster firing so that pressure build-up before the low-pressure valve stays within its operational range. Hence, the use of electric propulsion as an attitude control system alleviates the problem of fuel leaks and one of the major candidates of systematics on the Pioneer probes can be eliminated allowing us to assume $\Delta a_{\mathrm{leak}} = 10^{-11}\,\mathrm{m/s^2}$ for both mission concepts under consideration. For FEEP's the leakage rate would be reduced by at least one order of magnitude. We will however not include this option in our considerations as the low thrust generated by FEEP's would have considerable consequences in the design of the entire mission as already pointed out.

Out-gassing from the main structure of the spacecraft will in general not play a big role in the error budget. This is mainly due to the fact that the probe will already have travelled for a considerable time before the test of the Pioneer anomaly will be performed. Hence nearly all out-gassing will have taken place when the probe was closer to the Sun. A more important source of out-gassing could however be the RTG's of the spacecraft. In general the α-decay reaction in RTG's produces helium which will evaporate from the spacecraft. The decay of 1 kg of $^{238}\mathrm{Pu}$ produces approximately $4.2 \times 10^{-12}\,\mathrm{kg/s}$ of helium. Assuming an efficiency of 40 W/kg (e.g.

38.3 W/kg for the GPHS RTG used on Cassini) for the generation of electrical power we obtain a helium flow rate per generated watt of electric power of $\dot{M}_\alpha/P = 1.1 \times 10^{-13}$ kg/Ws. Furthermore it is reasonable to assume that the helium has reached thermal equilibrium before it flows out of the RTG's.[‡] Then its average velocity is given by $v_\alpha = \sqrt{3kT/m_\alpha}$, where k is Boltzmann's constant, m_α is the mass of a helium atom and the temperature of the RTG will typically be about $T = 500$ K. Hence the out-stream velocity of the helium will be $v_\alpha = 1.7 \times 10^3$ m/s. Assuming that all helium flows out unidirectionally, and taking into account the values given in table 1, we may work out the magnitude of the acceleration for the two spacecraft designs. In particular for missions, which have a nuclear electric propulsion system, the expulsion of helium can make an important contribution and its recoil effect on the spacecraft needs to be taken into consideration. This is done most easily by placing the pressure relief valves of the RTG's in a way that no net force results along the spacecraft's spin-axis results. We assume that the uncertainty in the acceleration due to helium out-gassing can be constrained to 2% of its worst case value, which corresponds to a placement of the valve perpendicular to the spin axis at a precision of 1 deg. For the planetary exploration missions this results into an uncertainty of $\sim 4.2 \times 10^{-12}$m/s^2 and for the piggy-back concepts we find an uncertainty of $\sim 2.1 \times 10^{-12}$m/s^2.

Heat

Heat is produced and radiated from the spacecraft at various points. The dispersion of heat, necessary to maintain the thermal equilibrium in the spacecraft, produces a net force on the spacecraft whose magnitude per Watt of non isotropically radiated heat is 3×10^{-9} N.

The heat generated in the main structure of the spacecraft will in general be of the order of a few 100 W. Assuming the above advocated spin stabilisation of the craft, the thermal radiation perpendicular to the spin axis of the satellite will average out over one rotation. Hence the radial component of thermal radiation does not contribute to the error budget for the measurement of a putative near constant, i. e. very low-frequency, acceleration. Placing the radiators so that the heat they dissipate does not produce a net force along the spacecraft axis, the contribution on the radiation force of heat can be reduced to a few Watts. Note, that the avoidance of reflections is much superior to the precision modelling of the thermal radiation characteristics of the spacecraft because the effect of surface deterioration during the journey is difficult to model. Thus the avoidance of reflections by restricting the opening angle of radiators is mandatory for a precision test of the Pioneer anomaly. The radiation from other surfaces of the spacecraft can be monitored to some extent by measurements of the surface temperature. This option is discussed below for the case of the RTG's. We will therefore assume as a spacecraft design requirement that radiators are positioned in such a way as to reduce the total force due to the radiated heat along the spacecraft spin axis to a fraction of the Pioneer anomaly, we will set $\Delta a_{\rm bus} = 1 \times 10^{-11}$ m/s^2.

By far the bigger source of thermal radiation are the RTG's, necessary to power the spacecraft systems. In particular if one chooses an electrical propulsion system the thermal heat to be dissipated from the RTG's may easily reach 10 kW for the exploration paradigm[42]. In principle an anomaly caused by RTG heat can be distinguished from other sources because it will exponentially decay

[‡]Actually, the helium plays an important role for thermal conduction in the RTG. We are grateful to M. M. Nieto for this information.

with the 87 years of half-life of the Plutonium, which would result in a change of approximately 8% in 10 years. In the case of the Pioneer spacecraft however, the disturbance by attitude control manoeuvres were so large that no reliable determination of a possible slope of the anomaly could be performed. For a new mission, in which gas leaks are well under control, a reliable measurement should however be possible. Nevertheless it is desirable to have an independent upper limit on the effect of RTG heat so that a reliable estimate can be given on this effect for each interval between attitude-control manoeuvres.

Hence it is preferable to reduce forces due to non-isotropic heat-emission from the RTG to the level of a fraction of expected anomaly. To accomplish this RTG heat must be dissipated fore–aft symmetrically and reflections from the spacecraft should, again, be avoided. This may be simply achieved by putting the RTG's on long booms or reduce their view factor towards other components of the spacecraft by a more intricate design. In case of the POP spacecraft, e. g., the choice was made to put the RTG's on short booms at the back of the spacecraft, inclined 45 deg to spin axis[42]. This design sufficed to reduce the RTG's view factor, but saved from the need to have foldable booms to meet the size constraint of the launch fairing. In combination with a detailed model of the radiation characteristics this will reduce any unmodelled directional heat radiation due to asymmetry to affordable values.

More troublesome is the effect of possible material degradation on the radiation characteristics of the RTG's. During a typical mission the antenna facing side of the RTG's will be exposed to Solar radiation almost permanently whereas the other side of the RTG's lies in shadow for nearly all of the mission. Hence one can expect a very asymmetric degradation of the emissivity and absorptance of the RTG's. Whereas it would be difficult to detect which part of the RGT's surface degrades faster – most likely it would be the sun-facing side – one can monitor the overall degradation of the emissivity ϵ of the RTG by monitoring its temperature T at selected points.

To demonstrate this we consider as a simplified model a cylindrical RTG, with the cylinder axis perpendicular to the spacecraft–Sun direction. Furthermore we assume perfect thermal conductivity of the RTG so that all of its surface is at the same temperature. We first derive a relation between the temperature and the emissivity change and then a relationship between the resulting change in acceleration and the emissivity change. We then show how under certain assumptions also temperature and acceleration can be directly related.

The azimuth angle ψ of the cylinder is measured from the Sun-pointing direction. Using the Stefan-Boltzmann law the relation between the total radiated power, $P_{\rm tot}$, the emissivity per angle $\epsilon(\psi) = \epsilon_0 + \Delta\epsilon(\psi)$ and the temperature of the RTG is given by

$$P_{\rm tot} = \frac{\mathrm{const.}\, T^4}{2\pi\epsilon_0}\int_0^{2\pi} [\epsilon_0 + \Delta\epsilon(\psi)]d\psi\,.$$

Since the thermal power produced by the RTG is well known from the amount of Plutonium in it the temperature of the RTG is directly related to change of emissivity $\Delta\epsilon$. Indicating with T_0 the temperature of the RTG when $\Delta\epsilon(\psi) = 0$, we have:

$$T = T_0 \left(\frac{2\pi\epsilon_0}{2\pi\epsilon_0 + \int_0^{2\pi}\Delta\epsilon(\psi)d\psi}\right)^{1/4}\,.$$

On the other hand the power per angle is related to the total radiated power by

$$P(\psi) = P_{\rm tot} \frac{\epsilon_0 + \Delta\epsilon(\psi)}{\int [\epsilon_0 + \Delta\epsilon(\psi)]\, d\psi}\,. \tag{1}$$

The effective asymmetric power radiated along the spin axis of the craft is given by

$$P_a = \int_0^{2\pi} P(\psi) \cos(\psi)\, d\psi\,. \tag{2}$$

Inserting Eq. (1) into Eq. (2) and expressing the acceleration a_ϵ induced by the change in emissivity we obtain

$$a_\epsilon = \frac{P_{\rm tot}}{M_{\rm S/C} c} \frac{1}{\int [\epsilon_0 + \Delta\epsilon(\psi)]\, d\psi} \int_0^{2\pi} \cos(\psi) [\epsilon_0 + \Delta\epsilon(\psi)]\, d\psi\,. \tag{3}$$

In general there will be no unique relation between T and a_ϵ because the quantities depend on different integrated functions of the emissivity. Nevertheless a relation can be established if one makes some reasonable model assumptions. To illustrate this we consider an RTG which has an original emissivity of $\epsilon_0 = 1$ and we model the emissivity change with the simple relation:

$$\Delta\epsilon(\psi) = -\Delta\epsilon_{\rm max} \cos(\psi)\,, \quad \text{for } |\psi| \leq \pi/2\,,$$

where $\Delta\epsilon_{\rm max} > 0$ is the absolute value of the change of reflectivity in the Sun pointing direction. In this case the deceleration of the spacecraft is given by

$$\Delta a_\epsilon = -\frac{P_{\rm tot}}{4 M_{\rm S/C} c} \Delta\epsilon_{\rm max}\,.$$

The temperature after the degradation of emissivity is then related to the temperature at nominal emissivity T_0,

$$T = \frac{T_0}{4\pi}\, \Delta\epsilon_{\rm max} + O(\Delta\epsilon_{\rm max}^2)\,.$$

We obtain the final relation

$$a_\epsilon = -\frac{\pi P_{\rm tot}}{c M_{S/C} c} \frac{T}{T_0}\,. \tag{4}$$

Consequently we find for the acceleration uncertainty Δa_ϵ in dependence of the uncertainty in temperature ΔT

$$\Delta a_\epsilon = \frac{\pi P_{\rm tot}}{c M_{S/C} c} \frac{\Delta T}{T_0}\,.$$

For an RTG the nominal temperature is $T_0 \sim 500\,$K. Hence assuming that we monitor the RTG temperature at a precision of 0.1 K and assuming the above degradation model we would have an uncertainty in the anomalous acceleration of $\Delta a_\epsilon = 2.8 \times 10^{-11}\,{\rm m/s^2}$ for the exploration scenario and $\Delta a_\epsilon = 1.4 \times 10^{-11}\,{\rm m/s^2}$ for the piggy-back scenario.

A realistic model of the RTG will be considerably more complicated. On the one hand this comes form the more involved geometry of the radiator fins. On the other hand the power of the Solar radiation impinging on the RTG's will be considerably higher – at least at the beginning of the measurement period – than the effect of the degraded emissivity. Hence the model will have to include also the absorptance and to account for a non-uniform temperature of the RTG and the Yarkovsky effect (cf. Cruikshank[49] or Peterson[50] for an overview of the Yarkovsky effect). These are, however, mainly numerical complications and it is always possible to develop a refined version of Eq. (4) so that the uncertainty of the RTG temperature measurements may be related to the uncertainty of the derived acceleration. In particular there is no danger of mistaking a degradation or failure of thermocouples of the RTG for a change in emissivity because these effects are distinguishable by the accompanying decrease of electric power. In the following we will assume the acceleration levels found with our simple model, which may always be interpreted as a system design requirement, which, for the reasons explained above, is not difficult to meet.

Radio-beam radiation force

The increasing amount of data gathered by modern planetary observation instruments demands high data transmission capabilities. For missions to the outer Solar System like the ones discussed here this inevitably leads to high transmission powers for the telecommunication system, $\sim 50\,\mathrm{W}$. Hence the reaction of the spacecraft to the radio beam may easily reach the order of magnitude of the Pioneer anomaly. However this systematics can be constrained in a straightforward way. During the coast phase in which the Pioneer anomaly is to be tested the data volume generated on-board will be much smaller than at the final destination of the probe. Hence the transmission power can be reduced to a few Watts during the test. bringing the uncertainty in the transmission power for both mission paradigms down to less than 1 W. This would correspond to an acceleration systematics below $Deltaa_{\mathrm{Radio}} = 5 \times 10^{-12}\,\mathrm{m/s^2}$ for the planetary exploration mission and $Deltaa_{\mathrm{Radio}} = 2.2 \times 10^{-11}\,\mathrm{m/s^2}$ for the piggy-back spacecraft. These numbers might be even further reduced by changing the transmission power to different values during the measurement period and measuring the subsequent change of the spacecraft acceleration. In this way one could actually calibrate for the effect of the radiation beam.

Solar radiation pressure

The last major contribution to discuss in this context is the Solar radiation pressure. For the present level of analysis it is sufficient to discuss the effect of the Solar radiation force by considering the force for flat disk of the size of the spacecraft antennas covered with white silicate paint. To further simplify our consideration we restrict ourselves to specular reflection and neglect diffuse reflection and the Yarkovsky effect. Then we can express the acceleration induced by Solar radiation pressure[51] as:

$$\mathbf{a}_\odot = \frac{P_\odot}{c\,r^2}\frac{A_{S/C}}{M_{S/C}}(1+\eta)\cos^2\theta\,\mathbf{e}_A\,, \tag{5}$$

where we have used the fact that the tangential force arising from the partial specular reflection has no effect on the centre-of-mass motion of the spacecraft due to the spin stabilisation. Here η denotes specular reflectivity coefficient of the antenna, $P_\odot = 1367\,\mathrm{W\,AU^2/m^2}$ is the Solar radiation constant. Since the antenna is oriented to Earth the vector $\mathbf{e}_A$ is Earth pointing and the two vectors $\mathbf{e}_A$ and $\mathbf{e}_\odot$ only enclose a small angle θ for large heliocentric distances, i. e. in all mission options for most of the measurement phase (see below). The uncertainty of the acceleration due to Solar radiation $\Delta\mathbf{a}_\odot$ is by dominated by a possible change of the reflectivity properties of the

	planetary exploration $\Delta a/(10^{-11}\mathrm{m/s^2})$	piggy back $\Delta a/(10^{-11}\mathrm{m/s^2})$
fuel leaks	0.4	0.2
heat from bus	1.0	1.0
heat from RTG	2.8	1.4
RTG helium outgassing	2.7	2.0
radio beam	0.5	2.2
Solar radiation pressure	$149\,(R_\oplus/r)^2 cos^2\theta$	$268\,(R_\oplus/r)^2 cos^2\theta$
total	$7.4 + 149\,(R_\oplus/r)^2 cos^2\theta$	$6.8 + 268\,(R_\oplus/r)^2 cos^2\theta$

Table 2: Acceleration uncertainties for the two mission paradigms.

spacecraft. We assume $\Delta\eta/\eta_0 = 5\%$. The uncertainties of all other quantities are about an order of magnitudes smaller and can be neglected for our purposes. Hence we find from Eq. (5), for the acceleration uncertainty due to Solar radiation pressure

$$\Delta\mathbf{a}_\odot = \frac{P_\odot}{c\,r^2}\frac{A_{S/C}}{M_{S/C}}\cos^2\theta\mathbf{e}_A\,\Delta\eta\,. \tag{6}$$

The maximal value is taken for $cos\theta = 1$, i. e. when the spacecraft is in conjunction. For the planetary exploration scenario we find $\Delta a_\odot = 149\,(R_\oplus/r)^2\,10^{-11}$ and for the piggy-back concept $\Delta a_\odot = 268\,(R_\oplus/r)^2\,10^{-11}$. We see from these numbers that it would be extremely difficult to be able to detect any anomaly on the spacecraft acceleration at distances smaller than 3.1 AU for the piggy-back and 1.9 AU for the exploration mission. At this distance the uncertainty on the Solar pressure force model would be greater than one third of the putative anomaly.

Summary of Systematics

In the previous section we have discussed the major sources of acceleration systematics for the two non-dedicated concepts under discussion and we have determined some intervals in which each of the disturbances is contained. The numerical results are summarised in table 2. For a spin stabilised craft all acceleration uncertainties act along the rotational axis of the spacecraft.

The sources of acceleration which were identified are uncorrelated – at least to the level of the modelling performed – and the overall acceleration due to systematic is therefore bounded by the value $\Delta a = \sum_i \Delta a_i$ that returns $\Delta a = 7.4 + 149\,(R_\oplus/r)^2 cos^2\theta$ for the exploration mission and $\Delta a = 6.8 + 268\,(R_\oplus/r)^2 cos^2\theta$ for the piggy-back spacecraft. This would only, when sufficiently far from the Sun, allow to determine the anomaly to a precision of 10 %, which is approximately one order of magnitude worse than the error-budget presented by Nieto et al.[35] for his Yo-Yo like dedicated spacecraft. The actual accuracy to which an anomalous acceleration can be determined, will also strongly depend on its direction. Since all error sources will cause an acceleration purely along the spin axis of the spacecraft. they will be competing with an Earth pointing anomaly, which would most likely be an effect on the radio signal. When studying the capabilities of the mission to discriminate the direction of the anomaly the systematic errors do not influence the result because their direction does not change and their magnitude has a gradient, which cannot be confused with a direction dependent modulation.

Summary on spacecraft design

From the goal to minimise the uncertainties in conventional accelerations we have arrive at several design requirements for our spacecraft: Spin stabilisation of the spacecraft seems mandatory in order to reduce the the number of attitude control manoeuvres on the spacecraft and ensures that all systematic accelerations are pointing along the spin axis of the craft. This effectively eliminates the effect of systematics on the determination of the direction of a putative anomaly (see below). For the exploration scenario spin axis stabilisation is most practically only chosen during the coast phases of the mission. An electric propulsion system turned out to be the most promising option to reduce the amount acceleration systematics from propellant leakage although an electric propulsion system has the disadvantage that due to its high power consumption it considerably increases the amount of heat generated on-board the spacecraft. The major source of asymmetric thermal radiation from the craft are the RTG's. The heat systematics can be constrained to a sufficient degree by monitoring the Temperature of the RTG's. Furthermore the view factor of the RTG's from the spacecraft bus and the antenna should be made as small as possible in order simplify reduce radiation backscattering and simplify the modelling. In order to constrain the systematics induced by the radio transmission beam two possibilities arise. The first one is to lower the transmission power during the measurement phase. An alternative consist of directly calibrating the Radio power to the level of $\sim 1\,W$ by measurement from a radio observatory. While the requirements imposed on the spacecraft design make it necessary that the spacecraft is already designed with the goal to test the Pioneer anomaly under consideration the modifications suggested come at no increase in launch mass and at no increase in risk. In particular the goal to test the Pioneer anomaly is compatible with the constraints of a planetary exploration mission.

Measurement strategies

Tracking methods

A mission to test the Pioneer anomaly has to provide three types of information. It must monitor the behaviour of the tracking signal for an anomalous blue shift, it must be able to detect an anomalous gravitational force acting on the spacecraft and it must also be capable of detecting an anomalous non-gravitational force on the spacecraft. We briefly review the available tracking methods to explain how their combination allows an unambiguous discrimination between the various possible causes of the anomaly (see Thornton and Border[52] for an introduction to tracking methods).

In sequential ranging a series of square waves is phase modulated onto the uplink carrier. The spacecraft transponds this code. The ground station compares the currently transmitted and the received part of the signal and determines the round-trip time from the comparison. Since the modulated signal is recorded and compared in order to obtain the distance from the spacecraft to the ground station the information obtained relies on the group velocity of the signal. The group velocity is influenced by the interplanetary plasma which acts as dispersive medium but not by gravitational effects which are non-dispersive. We assume a range error of 0.6 m at 1σ confidence level in our analysis[52].

Doppler tracking uses a monochromatic sinusoidal signal. The signal is sent to the spacecraft and is coherently transponded back to Earth. Both the phases of the outgoing signal and of the incoming signal are recorded. Since the frequency of the wave is the derivative of the phase, the frequency change between the outgoing and incoming wave can be determined and the relative velocity of the spacecraft and the tracking station can be inferred. The position is then obtained by integrating the observed velocity changes to find the distance between the spacecraft and the tracking station. The Doppler data are sensitive to other phase shifting effects such as the frequency shift by the interplanetary plasma and to a gravitational frequency shift. For long integration time the Doppler error is usually dominated by plasma noise, which typically leads to an error of approximately 0.03 mm/s at 1σ confidence level[52,53].

The simultaneous use of both tracking techniques allows for a correction of charged medium effects because for a signal, which propagates through a charged medium, the phase velocity is increased whereas the group velocity is decreased by the same amount[54]. The comparison of the Doppler and ranging measurements in order to determine Plasma effects has important benefits for non-dedicated test of the Pioneer anomaly because it allows a determination of the errors induced by the charged interplanetary medium without requiring dual frequency capabilities and is thus a considerable mass saver. Since the information of the sequential ranging relies on the group velocity of the signal and the information of the Doppler tracking relies on the phase velocity of the carrier, the usage of both ranging methods also allows to distinguish between a real acceleration of the spacecraft and an anomalous blue shift. Whereas a real acceleration would show up in both data, the frequency shift would only affect the Doppler signal, which is sensitive to changes in the phase velocity of the wave, but not to the sequential ranging signal which measures the group velocity.

Both, Doppler tracking and sequential ranging, are primarily sensitive to the projection of the spacecraft orbit onto the Earth–spacecraft direction. In order to characterise a putative anomaly it is however crucial to determine its direction. The three major candidate directions are Sun-directed along the velocity vector of the spacecraft and Earth-directed, the first two of which induce an annual modulation of the tracking signal. Unfortunately, we will see below that the annual modulation term is suppressed with at least the ratio between the spacecraft–Earth distance and the Earth–Sun distance. Hence for large heliocentric distances the magnitude of the modulation will rapidly drop to a few percent making it extremely difficult to detect.

In view of this problem it would be beneficial to obtain independent information on the motion of the spacecraft orthogonal to the line of sight. This information is in principle provided by Delta differential one-way ranging (ΔDOR). Differential one-way ranging determines the angular position of a spacecraft on the sky by measuring the runtime difference of a signal from the spacecraft to two tracking stations on Earth. Assuming that the rays from the spacecraft to the two stations are parallel to each other the angle between the spacecraft direction and the baseline connecting the two stations can be determined from the runtime difference. In ΔDOR the accuracy of this method is further improved by differencing the observation of the spacecraft from that of an astronomical radio source at a nearby position in the sky. The typical accuracy achievable with ΔDOR is 50 nrad at 1σ confidence level[52]. An improvement of accuracy by two orders of magnitude in angular resolution would be achievable if the next-generation radio-astronomical interferometer, the Square Kilometre Array could be used for the tracking[55]. However this observatory is not likely to be completed for the launch dates under consideration. Hence we do not include this enhanced capabilities in our analysis.

Tracking observables for the Pioneer anomaly test

The capabilities of the three tracking techniques for a specific trajectory are easily evaluated numerically by determining after which time a detectable deviation from the nominal trajectory has accumulated. For the present investigation we are however interested in the inverse problem of deriving the criteria that a trajectory has to fulfil to be well suited for an investigation of the Pioneer anomaly. For trajectories in the outer Solar System these criteria can be established from a simple linearised tracking model, which yields closed expressions for the performance of the different tracking techniques and describes the trajectory dependence of the tracking sensitivity in terms of a few trajectory parameters. The analysis considers the tree major candidates for the anomaly identified above: a Sun-pointing acceleration, an acceleration pointing in the opposite direction as the velocity vector of the spacecraft, and a blue shift of the radio signal.

Since the measurement of the Pioneer anomaly will have to be conducted by a mission to the outer part of the Solar System the perturbation on the position vector is well described, for our purposes, by the simple equation:

$$\ddot{\mathbf{s}}^* = \mathbf{a}^* \,,$$

where $\mathbf{s}^*$ is the difference between the position $\mathbf{r}$ of a spacecraft not affected by the anomaly and the position ρ of a spacecraft affected by the anomalous acceleration $\mathbf{a}^*$. In fact we may write (see Bate[56]) the full equation in the form:

$$\ddot{\mathbf{s}}^* + \frac{\mu}{r^3}\left[\left(\frac{r}{\rho}\right)^3 - 1\right]\mathbf{r} + \mu\frac{\mathbf{s}^*}{\rho^3} = \mathbf{a}^*$$

Note that this holds also for non-Keplerian $\mathbf{r}$ whenever the non-gravitational modelled forces may be considered state independent (as is the case of the systematic acceleration considered here). Evaluating how long it takes for the second and third term to grow within two orders of magnitude from the magnitude of $\mathbf{a}^*$ we find a time span of the order of 90 days at Jupiter distance. This means that the simple solution,

$$\mathbf{s}^* = \int_0^t \int_0^t \mathbf{a}^*(t')dt' + \mathbf{s}_0^*$$

can be used to gain insights on the capabilities of the various tracking techniques for a test of the Pioneer anomaly. In the following we will use it to determine the time it takes for a given anomalous acceleration to become visible in the tracking data and the information that these tracking measurements may give us on the anomaly's direction. We consider our spacecraft as lying in the ecliptic plane and we consider, without loss of generality, a uniform circular motion of the Earth. A visualisation of the geometry for this two-dimensional model is displayed in figure 1. We perform the calculation for an anomalous acceleration $\mathbf{a}^*$ of constant magnitude, as indicated by the Pioneer data, and fixed inertial direction (so that we need to determine only one angle to characterise it). We integrate $\mathbf{a}^*$ twice over a time interval t to obtain the anomalous

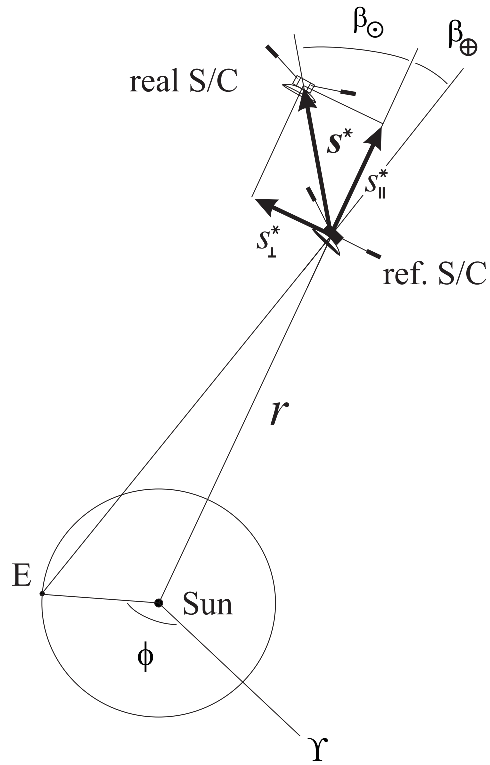

Figure 1: Tracking of the anomaly in the ecliptic plane.

velocity change $\mathbf{v}^*$ and position change $\mathbf{s}^*$. Then we project the anomalous velocity change and position change onto the Earth–spacecraft vector. The change in the geocentric angular position of the spacecraft on the sky, $\alpha^*_\oplus$, is obtained from the component of s^* perpendicular to the Earth–spacecraft direction through the relation $\alpha^*_\oplus \simeq \frac{s^*_\perp}{s}$. We get

$$v^*_\parallel = a^* t \cos[\beta(t)] \tag{7}$$

$$s^*_\parallel = \frac{a^*}{2} t^2 \cos[\beta(t)] \tag{8}$$

$$\alpha^*_\oplus = \frac{a^*}{2s} t^2 \sin[\beta(t)]\,, \tag{9}$$

where β is the angle between the putative anomaly direction and the Earth–spacecraft vector. The three equations above estimate the effect of a constant anomalous acceleration on the tracking observables. It will turn out convenient to express the angle β as the sum of the angle between $\mathbf{a}^*$ and the Sun–spacecraft vector and the Earth–spacecraft vector and the Sun–spacecraft vector $\beta_\oplus$ (cf. figure 1),

$$\beta = \beta_\odot + \beta_\oplus\,.$$

The angle $\beta_\oplus$, in turn, can be expressed through the mean anomaly of the Earth, ϕ, the Sun–spacecraft distance r and the Earth–Sun distance $r_\oplus$. Then we have

$$\beta_\oplus \approx \arcsin\left(\frac{r_\oplus \sin(\phi)}{r}\right) \approx \frac{r_\oplus}{r} \sin(\phi)\,, \tag{10}$$

where the approximation is valid for $r \gg r_\oplus$ (the error it introduces at Jupiter distance is less than 1%). Using the standard relation for the sine and cosine of the sum of two angles and using Eq. (10) we find to quadratic order in $\beta_\oplus$,

$$\begin{aligned}\sin(\beta) &= \sin(\beta_\odot) + \beta_\oplus \cos(\beta_\odot) - \frac{1}{2}\beta_\oplus^2 \sin\beta_\odot + O(\beta_\oplus^3) \\ &\approx \sin(\beta_\odot) + \frac{r_\oplus}{r}\sin(\phi)\cos\beta_\odot - \frac{1}{2}\left[\frac{r_\oplus}{s_\parallel}\right]^2 \sin^2(\phi)\sin\beta_\odot\,,\end{aligned}$$

$$\begin{aligned}\cos(\beta) &= \cos(\beta_\odot) - \beta_\oplus \sin(\beta_\odot) - \frac{1}{2}\beta_\oplus^2 \cos\beta_\odot + O(\beta_\oplus^3) \\ &\approx \cos(\beta_\odot) - \frac{r_\oplus}{r}\sin(\phi)\sin\beta_\odot - \frac{1}{2}\left[\frac{r_\oplus}{s_\parallel}\right]^2 \sin^2(\phi)\cos\beta_\odot\,.\end{aligned}$$

Inserting these results into Eqs. (7) to (9) and assuming $\phi(t) \equiv 2\pi t/t_\oplus + \phi_0$, we obtain

$$v_\parallel^* = a^* t\left[\cos\beta_\odot - \frac{r_\oplus}{r}\sin\phi_0\sin\beta_\odot\right] - a^* t^2\left[\frac{2\pi}{t_\oplus}\frac{r_\oplus}{r}\cos\phi_0\sin\beta_\odot\right] + O(r_\oplus/r)^2\,, \tag{11}$$

$$s_\parallel^* = \frac{a^*}{2}t^2\left[\cos\beta_\odot - \frac{r_\oplus}{r}\sin\phi_0\sin\beta_\odot\right] - \frac{a^*}{2}t^3\left[\frac{2\pi}{t_\oplus}\frac{r_\oplus}{r}\cos\phi_0\sin\beta_\odot\right] + O(r_\oplus/r)^2\,, \tag{12}$$

$$\alpha_\oplus^* = \frac{a^*}{2s}t^2\left[\sin\beta_\odot + \frac{r_\oplus}{r}\sin\phi_0\cos\beta_\odot\right] + \frac{a^*}{2s}t^3\left[\frac{r_\oplus}{r}\frac{2\pi}{t_\oplus}\cos\phi_0\cos\beta_\odot\right] + O(r_\oplus^2/sr^2)\,. \tag{13}$$

Note that the magnitude of the Doppler observable only grows linearly with time whereas the observables of ranging and ΔDOR grow quadratic in time. We also write the terms of order $O(r_\oplus/r)^2$ in the case $\cos\beta_0 = 1$ for $v_\parallel^*$ and $s_\parallel^*$ because these yield the leading order annual modulation for a Sun-pointing anomaly, $\beta_\odot = 0$,

$$O(r_\oplus/r)^2[v_\parallel^*] = -\frac{a^* t}{2}\left[\frac{r_\oplus}{s_\parallel}\right]^2 \sin^2(\phi(t)) \approx -a^* t\frac{1}{2}\left[\frac{r_\oplus}{r}\right]^2 \sin^2\phi_0 - a^* t^2\frac{\pi}{t_\oplus}\left[\frac{r_\oplus}{r}\right]^2 \sin(2\phi_0) \tag{14}$$

$$O(r_\oplus/r)^2[s_\parallel^*] = -\frac{a^*}{4}t^2\left[\frac{r_\oplus}{s_\parallel}\right]^2 \sin^2(\phi(t)) \approx -\frac{a^*}{2}t^2\,\frac{1}{2}\left[\frac{r_\oplus}{r}\right]^2 \sin^2\phi_0 - \frac{a^*}{2}t^3\,\frac{\pi}{t_\oplus}\left[\frac{r_\oplus}{r}\right]^2 \sin(2\phi_0)\,, \tag{15}$$

where the approximations hold for observation times small compared to 1 year. For α^* we need not consider higher-order terms because here the leading-order effect of the annual modulation is given by the second term of Eq. (13). Expressions analogous to Eqs. (11) to (15) hold for the systematic acceleration uncertainties determined above. One can determine after what time interval t a specific type of anomaly becomes detectable by demanding that the deviation in v, s or $\alpha_\oplus$ has to exceed the uncertainty induced by the sum of the tracking error and the error induced by systematic accelerations.

Measurement performance

The measurement performance is conveniently evaluated by splitting the change in a tracking observable f, f being one of $v_\|$, $v_\|$, $\alpha_\oplus$, induced by the anomaly into a constant component f^* and a time dependent component, δf^*, that is dependent on the direction, in which the anomaly acts.

First we consider the detectability of the anomaly without attempting to determine its direction. For this goal it is sufficient to consider the leading-order terms of Eqs. (11) to (13). In order for the anomaly to be detectable these terms have to exceed the measurement error. The measurement error in turn is given by the sum of the tracking error, and the uncertainty in the systematic accelerations, $f^* > f_\text{track} + \Delta f$. The two errors have to be added instead of taking their Pythagorean sum because the error induced by the uncertainty in the systematic accelerations is not of statistic nature.

We first consider the case of Doppler tracking, $v_\|^* > v_\text{track} + \Delta v$. Solving for the tracking time we find

$$t_v > v_\text{track} / \left(a^* \cos\beta_\odot - \Delta a\right) \,.$$

For sequential ranging, requiring $s_\|^* > s_\text{track} + \Delta s$, we obtain for the tracking time required to detect the anomaly

$$t_s > \sqrt{2 s_\text{track} / \left(a^* \cos\beta_\odot - \Delta a\right)} \,.$$

For ΔDOR the situation is even simpler because the uncertainty in the acceleration orthogonal to the Earth–spacecraft vector is suppressed compared to the uncertainty along the Earth–spacecraft vector by one power of $r_\oplus/r$. Hence we have to a good approximation $\Delta\alpha \approx 0$, which leads to the condition $\alpha_\oplus^* > \alpha_\text{track}$. Solving for t_α yields

$$t_\alpha > \sqrt{\frac{2 s \alpha_\text{track}}{a^* \sin\beta_\odot}} \,.$$

Both, sequential ranging and Doppler tracking easily detect the anomaly. For sequential ranging the time necessary to detect an anomaly of $a^* \cos\beta_\odot = 10^{-9}\,\text{m/s}^2$ is below one day at Jupiter and decreases to approximately 3/4 of a day for larger heliocentric distances. For Doppler tracking the time necessary to detect this anomaly is below 1.5 days at Jupiter distance and decreases to approximately 1.3 days further outward.

Despite of the suppression of systematic accelerations orthogonal to the spacecraft–Earth vector, ΔDOR cannot compete in performance with the other tracking methods. Even if we consider $a^* = 10^{-9}\,\text{m/s}^2$ and $\beta_\odot$ as large as 30 deg the detection of the anomaly takes 140 days already at 5 AU and rises to 370 days at 35 AU.

Significantly more challenging than the detection of the anomaly is the determination of its direction. Here we consider the detection of the three most plausible candidate directions: Sun pointing, along the velocity vector and Earth pointing. The case of an acceleration along the velocity vector is to a good approximation covered by the case of an acceleration having a fixed angle with the Sun–spacecraft vector because the change of the flight angle of the spacecraft along the trajectory will be very slow.

A Sun-pointing effect would be revealed by the variation of the tracking observables due to the Earth's rotation around the Sun. In order to detect unambiguously the annual modulation δf in a tracking observable f its modulation has to exceed the sum of the tracking error $f_{\rm track}$ and of the uncertainty in the annual modulation of systematic accelerations $\delta(\Delta f)$. The last contribution stems entirely from the uncertainty in the Solar radiation force, $\delta(\Delta f) = \delta(\Delta f_{\odot,\parallel})$ because all other accelerations are Earth pointing and do not show a modulation.

For Doppler tracking this requirement becomes

$$v_{\rm track} + |\delta(\Delta v_{\odot})| < \left|\delta v_{\parallel}^{*}\right| \, . \tag{16}$$

Using Eq. (6) this becomes for a Sun-pointing anomaly to leading order in $\beta_{\oplus}$

$$v_{\rm track} + \left|\delta\left\{\Delta\eta \frac{P_{\odot} A_{S/C}}{cr^2\, M_{S/C}} \int_0^t dt' \cos^2[\phi(t')]\right\}\right| < \left|-\frac{a^* t}{2}\left[\frac{r_{\oplus}}{r}\right]^2 \left(\sin^2[\phi(t)] - \frac{1}{2}\right)\right| \, , \tag{17}$$

where we have used Eq. (14) to determine the right-hand side of Eq. (17). In order to evaluate this expression without having to consider a specific trajectory we approximate the Sun–spacecraft distance during the measurement period by its value at the end of the measurement, which yields an upper limit on the detection time. Furthermore we set an upper limit of 6 months on the tracking time because this is the expected approximate time span between two attitude control manoeuvres. Longer time spans cannot be evaluated in search for the modulation because the attitude manoeuvres are expected to considerably degrade our knowledge of the orbital motion of the spacecraft. Putting this limit on the observation time and using the error budget determined above we find that the annual modulation is detectable by Doppler tracking up to 6.2 AU heliocentric distance for both the exploration spacecraft and for the piggy-back spacecraft.

Applying the same reasoning for sequential ranging one finds,

$$s_{\rm track} + |\delta(\Delta s_{\odot})| < \left|\delta s_{\parallel}^{*}\right| \, ,$$

which becomes for a Sun-pointing anomaly,

$$s_{\rm track} + \left|\delta\left\{\Delta\eta \frac{P_{\odot} A_{S/C}}{cr^2\, M_{S/C}} \int_0^t \int_0^t dt' \cos^2[\phi(t')]\right\}\right| < \left|-\frac{a^*}{4} t^2 \left[\frac{r_{\oplus}}{r}\right]^2 \left(\sin^2[\phi(t)] - \frac{1}{2}\right)\right| \, .$$

For both paradigms the annual modulation remains detectable in sequential ranging beyond 50 AU.

For ΔDOR the modulation term is suppressed compared to the constant term by a factor of $r_{\oplus}/r$. Considering the weak performance of ΔDOR for the constant term it is obvious that this method will not be capable of detecting any type of annual modulation.

Next we consider an anomaly which has a fixed angle $\beta_{\odot}$ with the Sun–spacecraft direction. The case is a good approximation to an anomaly pointing along the flight angle, an exact evaluation of which would require the consideration of a concrete trajectory. Again the only time-variable competing source of acceleration systematics is the uncertainty in the Solar radiation force. From the condition for detectability $|\delta f| > f_{\rm track} + \left|\delta(\Delta f_{\odot,\parallel})\right|$ we find

$$v_{\rm track} + \left|\delta\left\{\Delta\eta \frac{P_{\odot} A_{S/C}}{cr^2\, M_{S/C}} \int_0^t dt' \cos^2[\phi(t')]\right\}\right| < \left|-a^* t \frac{r_{\oplus}}{r} \sin[\phi(t)] \sin\beta_{\odot}\right| \, .$$

We assume $\beta_\odot = 15\,\mathrm{deg}$, which corresponds to the value that we will consider as the minimal flight angle for the trajectories in the following section. Limiting the tracking time to one year again we find that Doppler tracking can detect this type of anomaly until 22.9 AU for the exploration paradigm and until 22.4 AU for the piggy-back spacecraft.

With the same reasoning one finds for sequential ranging

$$s_{\text{track}} < \left| \left| -\frac{a^*}{2} t^2 \frac{r_\oplus}{r} \sin[\phi(t)] \sin\beta_\odot \right| - \left| \delta \left\{ \Delta\eta \frac{P_\odot A_{S/C}}{cr^2\, M_{S/C}} \int_0^t \int_0^t dt' \cos^2[\phi(t')] \right\} \right| \right| .$$

Again sequential ranging is capable of detecting the modulation term beyond 50 AU for both paradigms, which is more than sufficient for the mission types under consideration.

Naively one would expect constant values for v^* and s^* for an Earth pointing anomaly corresponding to a constant acceleration. This has for instance been stated by Nieto et al.[35]. An Earth pointing anomaly should however not correspond to a real force (which would put the Earth in the centre of the Universe) but rather to a blue shift of light, which in turn should be dependent on the light-travel time of the wave. Neglecting gravitational effects, the light travel time t depends on the Earth spacecraft distance as $t = s/c$. The orbit of the Earth around the Sun thus leads to an annual modulation of the apparent anomalous velocity of $\Delta v^* = 4r_\oplus a^*/c = 2 \times 10^{-6}\,\mathrm{m/s}$. This modulation is below the sensitivity of the current Doppler tracking capabilities. Hence to a good approximation an Earth pointing anomaly indeed shows no modulation. Moreover s^* should be zero for a blue shift of the radio signal. Of course an Earth-pointing anomaly results in $\alpha^*_\oplus = 0$.

In summary the situation presents itself as follows: Sequential ranging has proven itself to be the most powerful tracking technique for a verification of the Pioneer anomaly. In particular the discrimination between the candidate directions of a putative anomaly can be performed by sequential ranging during the whole length of the trajectories under consideration.

At first sight this result seems in contrast to the common wisdom that range data are usually inferior in quality to Doppler data[57]. However the standard situation, in which precision navigation is most relevant, is that of a planetary approach. In this case the gravitational field is rapidly changing along the spacecraft orbit and ranging data induce larger navigational errors than Doppler indeed. For the deep space situation of the Pioneer-anomaly test the gravity gradients are very low and the hence the reliability of sequential ranging data is much improved.

Doppler data will nevertheless be of high importance for the measurement. Only by the comparison of both data types, sequential ranging and Doppler, one can discriminate between a real acceleration and a blue shift of the radio signal.

ΔDOR showed to be of little use for a test of the Pioneer anomaly. In particular it cannot resolve the directionality of the anomaly. Hence, while it is certainly desirable to have occasional ΔDOR coverage during the Pioneer anomaly test to verify the orbit reconstruction of the spacecraft, ΔDOR cannot contribute to the precision determination of the anomaly.

From the analysis of the various tracking techniques we can also infer requirements on the trajectory of the spacecraft. For the reliable discrimination of an anomaly along the velocity vector of the spacecraft form other possible signatures of the anomaly a large flight angle is desirable. In particular, the lowest order modulation term signalling a velocity-pointing anomaly is proportional to the sinus of the flight angle. For flight angles close to $\beta_0 = 0$ the capabilities to distinguish an anomaly along the velocity vector from other candidate effects is suppressed $\sim r_\oplus/r$. Hence a large flight angle considerably improves the sensitivity to such an effect.

Up to now we have only briefly touched upon an effect, which could crucially degrade our measurement accuracy. The above estimates assume that the spacecraft remains undisturbed during the measurement period, necessary to detect the anomaly or its modulation. However this presupposes that no engine firings are necessary within the time span to detect the anomaly and furthermore that meteoroid impacts are rare enough to leave us with enough undisturbed measurement intervals to detect the modulation signals. Concerning thruster firings this condition is in fact fulfilled. The major disturbance torque in deep space will be the Solar radiation pressure. Even with a low rotation speed of 0.01 rpm, that we found beneficial for the exploration missions, the time span between thruster firings necessary to compensate for this disturbance will be in the order of months, leaving enough time to conduct precision measurements of the Pioneer anomaly. For the Pioneer 10 and 11 missions no disturbances due the gravitational field of asteroids could be noticed. Hence we can exclude this as a possible source of disturbance for our measurement. Analysis of the Pioneer tracking data also demonstrated that noticeable meteoroid impacts occur only at a frequency of a few per year. We are not trying to account for a continuous stream of impacts of small dust particles that are not visible as single events in the tracking data. Rather we consider such a stream as a putative source of the anomaly, which should in turn be recognised from its directionality.

Before turning to the actual discussion of the mission scenarios we briefly review the possibility of improving the test of the Pioneer anomaly by adding specific instrumentation to the mission in the next section.

Instrumentation options

An anomalous force on the spacecraft will show up as a deviation of the probe from its nominal trajectory calculated under the inclusion of all known forces. Such a deviation from the nominal trajectory will be detected by the radio tracking of the spacecraft. However the radio signal yields no information regarding the question if an anomalous force is of gravitational type or not. Such a distinction could in principle be accomplished with an accelerometer on-board the spacecraft because deviations of the spacecraft from a geodesic motion will be induced by non-gravitational forces, only.

Modern accelerometers like that to be used on-board ESA's GOCE mission[58] have a sensitivity of $3 \times 10^{-12}\,\mathrm{m/s^2}$ with a range up to a maximal acceleration of $3 \times 10^{-5}\,\mathrm{m/s^2}$. Hence, such accelerometers seem well suited to monitor the non-gravitational forces parallel to the rotational axis of the spacecraft, which are typically of the order of $10^{-9}\mathrm{m/s^2}$. Several open issues arise concerning the usefulness of accelerometers. Firstly accelerometers are not capable of detecting constant accelerations hence they can support other measurement strategies by monitoring variable acceleration noise, e. g. the thruster firings, but they cannot replace measurements by tracking. Secondly accelerometers have never been used in conjunction with a spinning spacecraft. Hence considerable developments would be necessary or the attempt to reduce the number of attitude manoeuvres by spin stabilisation would have to be given up in favour of reconstructing the thrust history from accelerometer read-out.

However we need not go into the details of such a measurement concept because for non-dedicated mission the use of high precision accelerometers seems excluded by weight constraints: high precision accelerometer assemblies weight typically in the order of 100 kg. which is to be compared to the 20 kg mass of the science payload of POP[42] or the overall wet mass of 180 to 200 kg for the piggy-back class spacecraft. Since accelerometers are not an option for a non-dedicated mission the determination between a gravitational and non-gravitational anomaly has to rely on the indirect means discussed previously.

In order to improve our understanding of disturbing forces generated by the space environment in the outer Solar System and make sure that they cannot contribute significantly to the Pioneer anomaly it is desirable to include a diagnostics package in the payload consisting out of a neutral and charged atom detector and a dust analyser. The mass of such a package would be approximately 1.5 kg.[47]

Trajectory design

We have already discussed how the introduction of a momentum dependence of the gravitational coupling could explain why the Pioneer anomaly does not show in the planets ephemerides. Even more straightforwardly, an amplification of the anomaly at high velocities could occur if matter on low-eccentricity orbits around the Sun cause a drag force (Note however that there does not seem to be enough dust available[5]). As a reflection of such possibilities, it is desirable to conduct the Pioneer anomaly test along a ballistic trajectory having a high radial velocity, i. e. a hyperbolic escape trajectory, rather than on a bound orbit. Otherwise, the choice of the inclination, of the argument of perihelion and of the longitude of the ascending node of this nearly Keplerian phase would not affect the test. In fact an explicit dependence of the anomalous force on the position of the spacecraft within the Solar System is highly improbable. This follows from the observation that the anomaly on both Pioneer probes does not change significantly with the position of the spacecraft along their orbits (a small change cannot be excluded due to the large error margin of the data) and that the trajectories of the two Pioneer head away from the Sun in approximately opposite directions and at considerably different inclinations thus making it obvious to conclude that if such a dependence exists, then it has to be as small as to be undetectable from the study of the Pioneer data. Also if the anomaly has its origin in a frequency shift of the radio signal its magnitude would probably be completely independent of the spacecraft motion. This would be the case for the explanation models relying on an influence of the cosmological constant on light propagation[20].

From the data of the Pioneer probes no precise determination of the direction of the anomalous force was possible. This mainly followed from the fact that Doppler tracking is able to determine the velocity of a spacecraft only in the radial direction. In particular, it was not possible to distinguish between the three major candidate directions of the anomaly: towards the Sun, towards the Earth and along the trajectory. The uncertainty in the on-board generated accelerations makes it therefore desirable to design the spacecraft trajectory in a way that facilitates the distinction between the candidate directions when analysing the tracking data. The result, discussed in the measurement performance section, is intuitive: a large flight angle is beneficial, which is unfortunately contradictory to the wish to have high radial velocity of the spacecraft. The flight angle can also be enlarged by conducting the Pioneer-anomaly measurement as far inward in the Solar system as possible. Unfortunately, the last requirement is conflicting with the goal to have the smallest possible systematics generated by Solar radiation pressure. A trade-off between these conflicting requirements has to be made on a case-by-case basis and is here discussed in a number of possible trajectories. As the Cosmic-Vision Programme of the European Space Agency refers to the decade 2015-2025, this will be used as a baseline for the trajectories here considered. Missions to Pluto, Neptune and Uranus are discussed separately from those to Jupiter and Saturn as the distance of these planets allow for a Pioneer anomaly test to be taken by the main spacecraft during the long travel.

POP class missions to Pluto, Neptune and Uranus

In this paragraph we discuss the possibility of using putative exploration missions to Pluto, Neptune and Uranus to perform the Pioneer anomaly test. We will first consider simple fly-by missions to these outer planets. These kind of missions are not to likely to happen as the scientific return of a fly-by is quite limited and it has already been exploited in several past interplanetary missions, which currently makes this kind of mission profile quite unattractive. We will therefore go one step further and consider orbiter missions exploiting Nuclear Electric Propulsion (NEP) for a final orbital capture. The trajectory baseline that is here considered is that of one sole unpowered gravity assist around Jupiter, many trajectories options and missions are of course possible to explore these far planets, see for example Vasile et al.[59], but a single Jupiter swing-by is probably the most plausible baseline in terms of risk and mission time. The purpose is that of showing that a Pioneer anomaly test would in general be possible, on these missions, on the vast majority of the possible trajectories. In the considered mission scenario the Pioneer anomaly test would be performed during the ballistic coast phase after Jupiter. As already discussed, a good trajectory, from the point of view of the Pioneer anomaly test, is an hyperbolic trajectory that has (during the test) a large flight angle γ (allowing to easily distinguish the velocity direction with the spacecraft–Earth direction) a long ballistic phase and a large Sun–spacecraft–Earth angle (allowing to distinguish between the Earth direction and the Sun direction). From standard astrodynamics we know that along a Keplerian trajectory we have:

$$\sin\gamma = \frac{\sqrt{p}}{r\sqrt{\frac{2}{r}-\frac{1}{a}}},,$$

where p is the semilatus rectum and a the semi-major axis of the spacecraft orbit. It is therefore possible to evaluate the angle γ at whatever distance from the Sun by knowing the Keplerian osculating elements along the trajectory after Jupiter. In particular we note that given a value for p, highly energetic orbits (i.e. fast transfers) lead to smaller values of the angle γ. This could lead to prefer a slower orbit. However a low velocity could lead to a smaller value of the anomaly (see above). The long ballistic arc requirement (an issue for orbiter mission baselines) goes in the same direction, in-fact the on-board propulsion (assumed to be advanced electric propulsion) could start to break the spacecraft much later in a slower trajectory (the arrival C3 on a Lambert arc gets smaller in these missions for longer transfer times). The third criteria added to design a mission that also tests the Pioneer anomaly is that of having a large Sun–spacecraft–Earth angle during the test phase. Trivially this implies that the test has to start as soon as possible after the Jupiter Swing-by not allowing for a long thrust phase immediately after the swing-by as would be required by optimising some highly constrained trajectory for low-thrust orbiter missions. One could therefore conclude that the first two requirements have an impact on the choice of the initial trajectory selection (that is mainly influencing the transfer time), whereas the third one has an impact on the subsequent steepest descent optimisation that determines the planet capture strategy. In order to assess the impact of these requirements on the trajectory design we therefore start looking at a multi-objective optimisation of an Earth-Jupiter-Planet fly-by mission assuming pure ballistic arcs and an unpowered swing-by (an orbiter mission may be later obtained by running a local optimiser having the pure ballistic trajectory as an initial guess). The multi-objective optimisation was tailored at minimising the $C3$ at Earth departure and minimising the mission duration (this parameter is directly related to the flight path angle and ballistic arc length). The Earth departure date t_e the Jupiter swing-by date t_j and the Planet arrival date t_p were left free to change with the departure date being constrained to be within the Cosmic Vision launch window and the arrival date being forced to be less than 2100. The Pareto fronts were built by using a preliminary version of DiGMO (Distributed Global Multi-objective Optimiser[60]) a tool being developed within the European Space Agency by the Advanced Concepts Team, able to perform distributed optimisations with a self-learning resource allocation strategy. Differential evolution [61] was used as a global optimisation algorithm until a stall of 100 generations triggered a steepest descent optimiser based on sequential quadratic programming. The final solution was then stored in the Paretian set only if not dominated by any other and discarded otherwise. All the solutions dominated by a new entry were discarded from the Paretian set. The objective function used was the following:

$$J = C3_{dep} + k(t_p - t_j)$$

with k being randomly chosen by the central server before assigning each task to the client machines[60].

Constraints (implemented as penalty functions) where considered on the departure date and on the arrival date,

$$t_e \in [2015, 2025]$$
$$t_p < 2100$$

on the Jupiter swing-by altitude ($h_p > 600000\,\mathrm{km}$) and on the inbound and outbound asymptotic velocities with respect to the Jupiter reference frame $\tilde{v}_{\mathrm{in}} - \tilde{v}_{\mathrm{out}} < 0.01\mathrm{km/s}$. The planet ephemerides used were the JPL DE405 in the range 2000-2100.

The results, visualised in figure 2, show two main optimal launch opportunities for the Earth-Jupiter-Pluto transfer: January 2015 and December 2016. The 2015 launches result in a slower first guess trajectory (from 17 to 27 years) with lower C3s (of the order of $87\,\mathrm{km}^2/\mathrm{s}^2$), whereas the 2016 window results in a shorter mission (from 11 to 15 years) with slightly higher C3s (of the order of 92-100 $\mathrm{km}^2/\mathrm{s}^2$). From a Pioneer anomaly test point of view the only trajectories that would not allow a good test are the very fast ones as the β angle would become as small as 15 deg already at 25 AU. All the others would be feasible and the anomaly test would affect only the further optimisation as it would require a long ballistic arc with no thrust phase immediately after Jupiter. This requirement is discussed later.

Similar results are obtained for the Neptune case (see figure 3). There are two optimal launch windows in the considered decade, January 2018 and February 2019. The first window allows for very low C3s (of the order of $75\,\mathrm{km}^2/\mathrm{s}^2$) and transfer times ranging from 14 to 40 and more years,

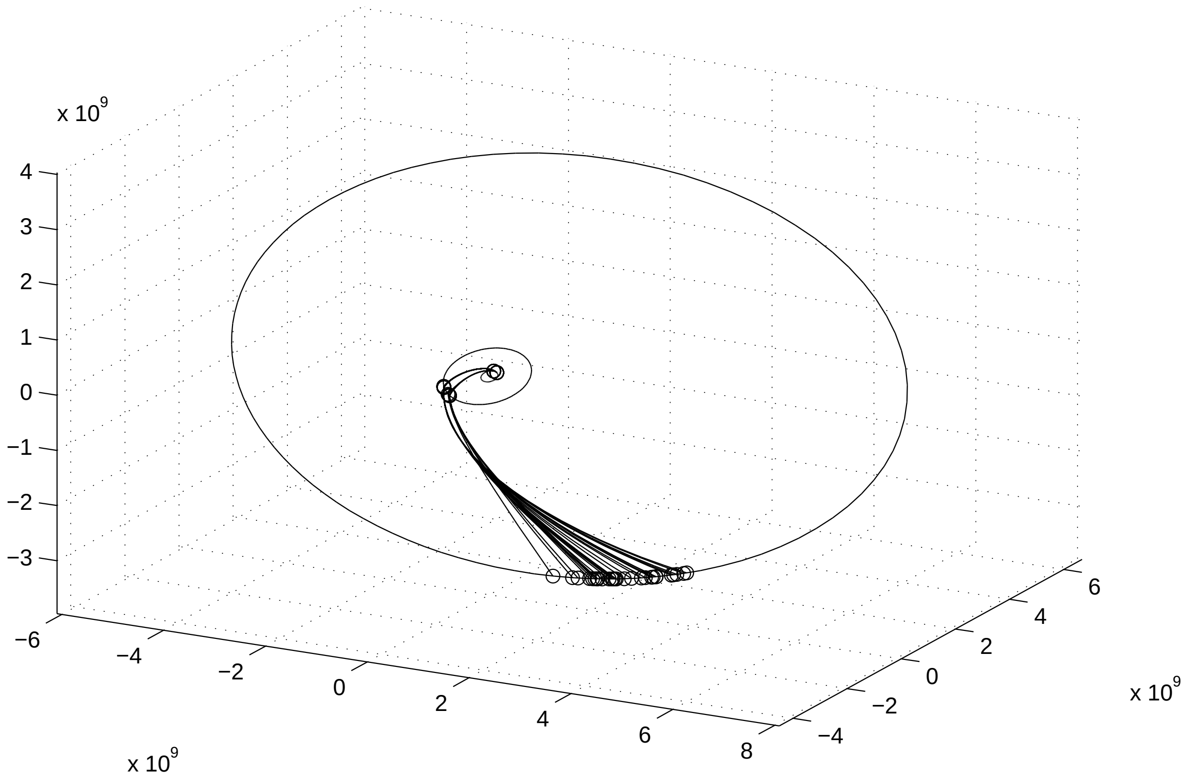

Figure 2: Paretian solutions for the Earth-Jupiter-Pluto mission within the decade 2015-2025.

whereas the second launch window is characterised by higher C3 values (ranging from 90 to 95 km^2/s^2) and shorter mission times (as low as 10 years). The requirement on the β angle is, in this case, satisfied by all the trajectories belonging to the found Pareto front.

The situation for Uranus missions, visualised in figure 4, is slightly more complex. Three main launch windows (and trajectory typologies) are possible. The first one, corresponding to a late Jupiter fly-by, is in March 2020 (repeating in April 2021) and corresponds to a C3 of roughly $81\,km^2/s^2$ (rising to $96\,km^2/s^2$ one year later) and to missions as short as 9 years. The other two are in December 2015 and December 2016 producing optimal first guess trajectories with C3s of the order of 78-79 km^2/s^2 and transfer times either very high (33 years) or of the order of slightly more than a decade. Due to the vicinity of the planet also in this case the value of the angle β is not an issue. We just observe that an hypothetical mission to Uranus exploiting one Jupiter fly-by would probably exploit the 2020 launch opportunity paying an augmented C3 cost of approximately $2\,km^2/s^2$ to buy several years of mission time. The conclusions of the preliminary

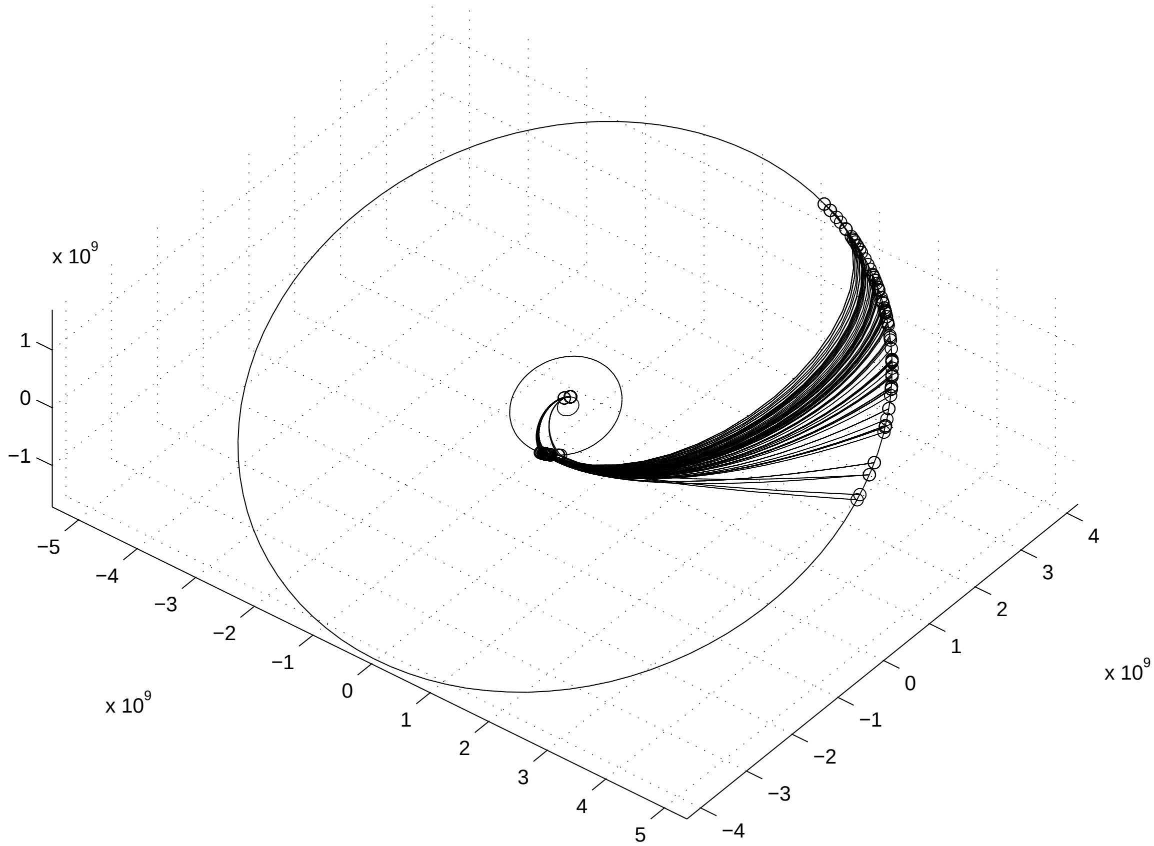

Figure 3: Paretian solutions for an Earth-Jupiter-Neptune mission within the decade 2015-2025.

multi-objective optimisation are are summarised in table 3.

Each of the trajectories belonging to the Pareto fronts might be modified to allow an orbiter mission by assuming some thrusters on board the spacecraft to steer it into a planet orbit. As discussed above, notwithstanding some concepts to navigate into deep space with Solar electric propulsion, it seems that the nuclear electric option is the most convenient and has to be used if we want to navigate in the outer regions of the Solar System. Starting from one of the trajectories of the Pareto-Front, if the launcher is able to provide all the C3 that is required and we do not apply heavy constraints the optimal trajectory will be ballistic up to the very last phase as shown in figure 5 for a Pluto case. If the problem is more constrained, for example if we add a departure C3 upper limit, then the ion engines need to be fired before and after Jupiter as shown in the other optimised trajectory. The firing immediately after Jupiter is necessary to assure that Pluto orbit is reached at the right time (This was the case for the POP trajectory, cf. [42]). In this case

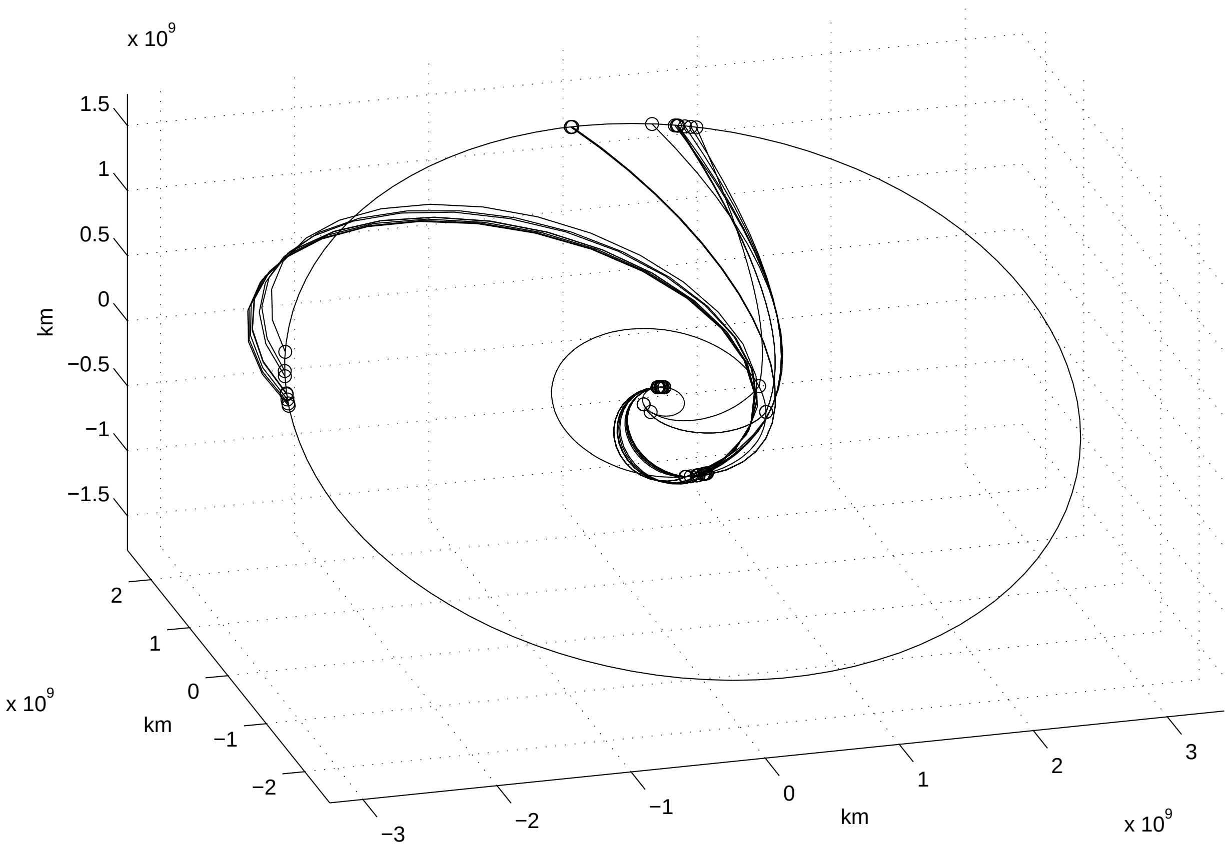

Figure 4: Paretian solutions for an Earth-Jupiter-Neptune mission within the decade 2015-2025.

a Pioneer anomaly test would return less scientific data because the thrusting phases could not be used for the characterisation of the putative anomaly. Adding as a constraint not to use the engines immediately after Jupiter, on the other hand, would introduce an increase in the fuel mass needed due to the late trajectory correction. This occurrence would hardly be accepted by the system designers and the Pioneer anomaly test would anyway be possible during the subsequent coast phase.

We may conclude that any trajectory of a fly-by or of an orbiter mission to the outer planets Pluto, Neptune and Uranus is likely to be suitable for a Pioneer anomaly test with no modifications, meaning that the three main requirements discussed would be fulfilled during a trajectory arc long enough to gain significant insights into anomaly.

Target Planet	Departure Date	Mission Duration (years)	Departure C3 (km^2/s^2)
Pluto	2015-Jan	17-27	87-88
Pluto	2016-Dec	11-15	92-100
Neptune	2018-Jan	14-40	74-75
Neptune	2019-Feb	10-12	90-95
Uranus	2020-March	9	81
Uranus	2021-April	7	96
Uranus	2015-16-Dec	12-14	79
Uranus	2015-16-Dec	28-33	79

Table 3: Pareto-optimal launch windows for fly-by missions to the various outer planets in the considered decade.

JIMO class missions to Jupiter and Saturn

A different situation occurs if we try to test the Pioneer anomaly exploiting a programmed mission to Jupiter or Saturn. In these cases the proximity of the planets to the Sun and the likely low energy of the transfer orbit would not allow for the test to be performed during the travel to the planet. A possible solution is that of designing a piggy-back spacecraft to be added as a payload to the main mission. We already presented a preliminary assessment of the dry mass of such a payload and we now discuss what the fuel requirement would be on such a spacecraft. In order to get an idea on what the mother-spacecraft trajectory would look like we simply considered the JIMO baseline and we performed an optimisation of a 2016 launch opportunity. The thrust was considered to be fixed and equal to 2 N for a spacecraft weighting 18000 kg. This was done to get some information on the switching structure of the thrust so that possible strategies of jettisoning could be envisaged. Final conditions at Jupiter do not take into account its sphere of influence. The optimised trajectory (visualised in figure 6) foresee a June 2016 injection in a zero $C3$ heliocentric trajectory and a rendezvous with Jupiter in May 2023. It is clear that the small piggy-bag should not affect at all the original trajectory, optimised for the main mission goals, and that it should be designed not to disturb the main spacecraft. A feasible solution seems to be to design a spacecraft capable to detach from the mother-spacecraft at the border of the arrival-planet's sphere of influence, navigate towards a powered swing-by of the target planet and put itself autonomously into an as-high-as-possible energy hyperbola. Some general estimates may then be made. We assume that the piggy-back is at the border of Jupiter's sphere of influence with zero $C3$. The gravity assist has to allow it to gain enough energy to have, in heliocentric frame, an hyperbolic trajectory. We also allow for a small flight angle γ at Jupiter. Under the assumption of a tangential burn at the pericentre (a more complete model such as that by Gobetz[62] should be used for a more detailed analysis) we may write for the required ΔV:

$$\Delta V = \sqrt{V_P^2(3 - 2\sqrt{2}\cos\gamma) + 2\frac{\mu_P}{r_{pP}}} - \sqrt{2\frac{\mu_P}{r_{pP}}}\,,$$

where V_P is the heliocentric velocity of the planet, μ_P its gravitational parameter and r_{pP} the pericentre of the incoming and outgoing hyperbolas.

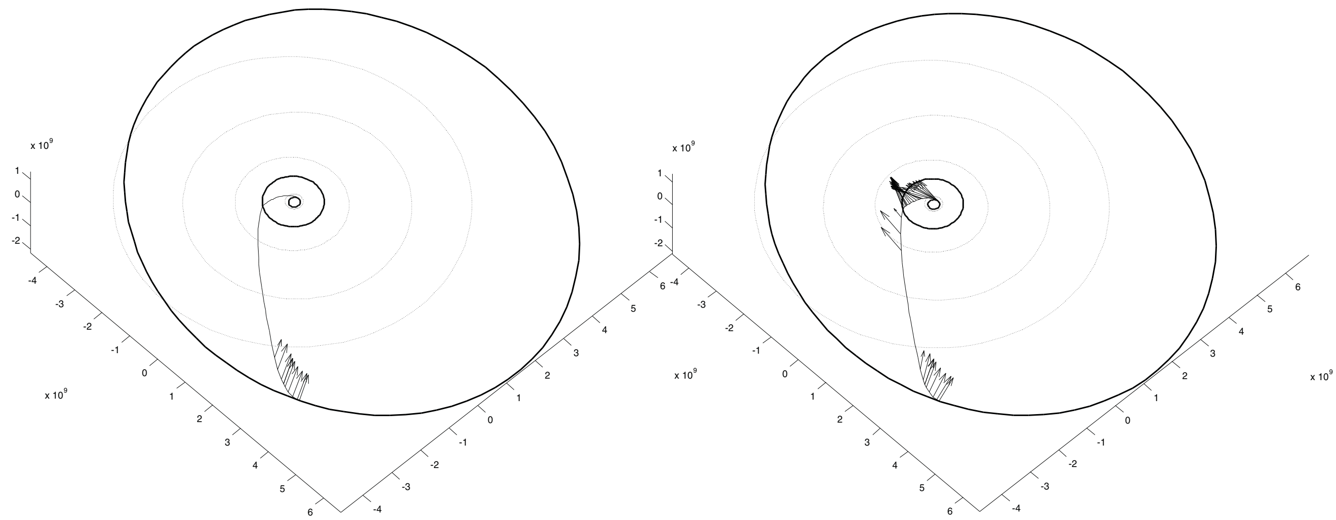

Figure 5: Optimised trajectories to Pluto with and without departure $C3$ constrains.

	γ (deg.)	0	7.5	15	22.5	30	37.5	45	52.5	60
Jupiter case	ΔV (km/s)	.7	.8	1.1	1.6	2.2	3	3.8	4.8	5.8
	$\frac{\Delta M}{M_0}$	.32	.37	.53	.84	1.3	2.2	3.5	5.6	8.9
Saturn case	ΔV (km/s)	.17	.2	.27	.39	.55	.76	1	1.3	1.6
	$\frac{\Delta M}{M_0}$	.071	.081	.11	.17	.24	.35	.48	.65	.86

Table 4: Piggy-back thrust requirements

Once the required ΔV is obtained it is easy to work out the ratio between the propellant mass and the spacecraft dry mass using the Tsiolkovsky equation:

$$\frac{\Delta M}{M_0} = e^{\frac{\Delta V}{I_{sp} g_0}} - 1\,,$$

where g_0 is the gravitational acceleration on the Earth's surface. Assuming the use of chemical propulsion for the powered gravity-assist ($I_{sp} = 260\,\mathrm{s}$) and putting a constraint on the gravity assist altitude of 600000 km in the Jupiter case and 40000 km in the Saturn case, the numbers in table 4 may be evaluated for the required ΔV and fuel to dry mass ratio. Due to the high pericentre required and due to its greater velocity the Jupiter case requires a higher propellant mass.

As a consequence the same spacecraft designed for a $\gamma = 15\,\mathrm{deg}$ Jupiter case is able, in the Saturn scenario, to go into a $\gamma = 35\,\mathrm{deg}$ trajectory. The resulting trajectories are illustrated in figure 6.

It is possible to see that the introduction of an angle γ greater than zero allows for trajectories that going inward and have better performances for a Pioneer anomaly test. They in-fact allow

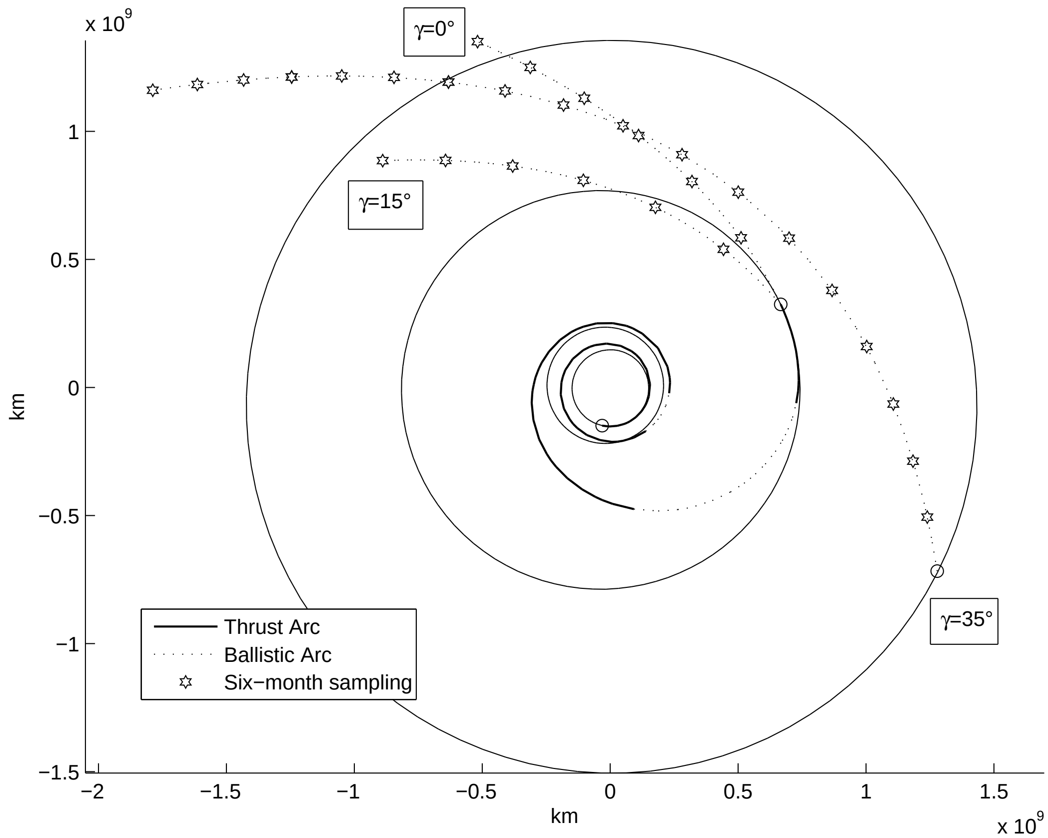

Figure 6: Piggy-back trajectory options.

for longer periods, in which the direction of the anomaly could be precisely measured, since the modulations in the tracking signal by the motion of the Earth, that enable the determination of the direction of the anomaly, are enhanced for low heliocentric distances. The cost in term of propellant mass can be evaluated from table 4.

Conclusions

We have analysed aspects of non-dedicated options for an investigation of the Pioneer anomaly. The considerations were focused on two likely scenarios for the exploration of the outer Solar System. First a low-mass low-thrust mission to Pluto, Neptune or Uranus. For this mission typology the Pioneer anomaly investigation is performed by radio-tracking of the exploration spacecraft. The other mission paradigm considered is that of a small piggy-back spacecraft, to be jettisoned from a large nuclear-reactor powered spacecraft to explore Jupiter or Saturn. The small spacecraft would be jettisoned from the mother-craft on the approach to its destination, would use the target planet of the mother-craft for a powered swing-by and subsequently perform the Pioneer anomaly investigation by radio tracking on a hyperbolic coast arc. Starting from a review of our knowledge about the effect and the models for its explanation, we have located a set of minimal requirements on the spacecraft design and trajectory.

The major prerequisite for a successful investigation of the Pioneer anomaly is minimal uncertainty in the systematic accelerations generated on-board and external to the spacecraft. On-board systematics arises mostly from attitude control manoeuvres, fuel leakage and waste heat. The suggested design responses to this are spin-stabilisation of the spacecraft during the measurement period and the reduction of the fuel leakage effects by use of electric propulsion. The effect of waste heat is shown to be alleviated by taking care of the spacecraft radiation characteristics, avoiding reflection of radiated heat on other parts of the spacecraft. Monitoring the temperature of the spacecraft at selected points can be used to track possible changes in its thermal-radiation characteristics. The dominant external acceleration systematics is located in the uncertainty in the determination of the Solar radiation force due to possible degradation of the optical properties of the spacecraft materials.

The implementation of these design requirements is similar in both mission paradigms. Whereas for the piggy-back craft a spinning mode with a high rotational velocity is most beneficial in order to achieve a high stability of the spacecraft against external disturbances a low rotational velocity is to be chosen for low-mass low-thrust exploration spacecraft.

For both mission classes the investigation of the Pioneer anomaly is performed by radio tracking. For the trajectories beyond Jupiter an anomalous force of the magnitude of the Pioneer anomaly can be treated in a linearised approximation without considering the back-reaction of the anomalous force on the spacecraft's orbital parameters, for periods of several months. This allows to treat both, an anomalous force and an anomalous blue shift, by the same formalism. Furthermore it allows to determine the effect of the Pioneer anomaly on the tracking data from a spacecraft in an analytic form, which has a simple dependence on the orbital parameters of the spacecraft.

Using the linearised model the time interval necessary for radio tracking to detect the presence of an anomaly is determined and the posibility of discriminating between different directions of the anomaly is investigated. The evaluation is done for the three most commonly employed tracking techniques, Doppler, sequential ranging and Delta-differential one-way ranging (ΔDOR). It is found that sequential ranging is performing best for the detection of the anomaly. ΔDOR is not capable of discriminating between different types of anomaly at all. For the trajectory selection it is found that a large flight angle eases the discrimination between the various candidate directions of the anomaly.

For the planetary exploration paradigm a multi-objective optimisation for low C3 and low transfer times for the various low-mass mission scenarios under the constraint of a minimal flight angle of $\beta = 15\,\mathrm{deg}$ was carried out. The Pareto front for launch dates in the time frame from 2015–2025 was determined. It was found that the constraint on the flight angle does not significantly restrict the launch opportunities. Hence a Pioneer anomaly test does not impose severe constraints on the choice of trajectory for low-thrust mission to the outer Solar System.

Also the trajectory options for piggy-back spacecraft to perform a Jupiter or Saturn swing-by were investigated. Particular attention was paid to the possibility of having a significant part of the test arc at relatively low heliocentric distances, where the determination of the direction of the anomaly is easier to accomplish. . This is achievable in particular for Saturn swing-bys where the absence of a radiation belt allows for considerably larger flight angle after the swing-by than for the Jupiter case.

For both mission paradigms the detection of the anomaly is possible during the whole measurement phase. Also the distinction between the three major candidate directions, towards the Earth, towards the Sun and along the velocity vector of the craft is possible from the modulation of the sequential ranging signal until well beyond 50 AU. Hence a non-dedicated test of the Pioneer anomaly offers the possibility to settle the controversy if the Pioneer anomaly is an indication of new physics or not.

There are however limitations in the scientific return from a non-dedicated mission. On-board systematics limits the precision for the determination of the magnitude of the anomaly to approximately 7% of the expected anomaly. While this is sufficient for its verification it is unsatisfactory for a precise characterisation. In particular a slope of the anomaly would most likely only be determined to first order. This would be hardly sufficient to determine unambiguously the physical law that might underly the Pioneer anomaly.

In conclusion non-dedicated missions can verify the existence of the Pioneer anomaly. They can furthermore determine the direction of a putative anomaly and provide some information about its gradient. The quality of their scientific return can however not compete with a dedicated mission based on the mother–child spacecraft formation currently favoured in the scientific community[40]. In view of the ongoing controversial discussion about the origin of the Pioneer it seems however more appropriate to take the more modest approach of using a non-dedicated mission to verify if the Pioneer anomaly is indeed an indication of a novel physical effect.

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