A Test for Linear Dependence of Stray Light in Junocam Image EFB03

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Abstract

The image EFB03 taken by Junocam during Earth flyby shows mostly stray light. Therefore EFB03 is particularly well-suited to analyse the structure of stray light in Junocam images. Describing the stray light of each framelet as a respective weighted sum of a fixed finite set of template framelets, with weights in $\mathbf{R}_{\mathbf{0}}^{+}$, would reduce the parameter space of possible stray light. A test for this assumption is described. Application of the test, however, points towards a more complex structure of the stray light in EFB03. [1]

[1]This document was typeset with LaTeX.

This article has been published as part of a post at `unmannedspaceflight.com`.

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1 Introduction

Images taken by Juno's Education and Outreach camera Junocam during the Earth flyby (EFB) in October 2013 are providing a first publicly available set of in-flight tests, similar to the images expected to be taken during Juno's Jupiter mission starting in mid-2016. So this article may be put into the context of [1, subsection 6.4], goal 3: "Provide data to the amateur image processing community and encourage them to produce a variety of products".

Junocam returned 17 images during Earth flyby, named EFB01 to EFB17. The image EFB03 taken by Junocam during Earth flyby shows significant stray light. It doesn't show many primary objects. Therefore EFB03 is particularly well-suited to analyse the structure of stray light in Junocam images. EFB03 consists of framelets of only one (the green) color channel. Let c be a set of columns, and r a set of rows, usually $c = \{1, \ldots, 1648\} \subset \mathbf{N}$, and $r = \{1, \ldots, 128\} \subset \mathbf{N}$. The article provides a test for the stray light $v : r \times c \to \mathbf{R}$ of each framelet being a finite weighted sum $v : (s,t) \mapsto \sum_i a_i x_i(s,t)$, with weights $a_i \in \mathbf{R}$, of a fixed finite set $\{x_i : r \times c \to \mathbf{R}\}$ of template framelets.

This approach may intuitively be motivated by the assumption of the stray light being caused by few small light leaks, together with the assumption, that small light leaks don't preserve much structure of the light source within one image. Assume the structure of the stray light being mostly determined by the almost constant physical structure of the camera.

To avoid motion blur due to the rotation of the Juno probe, Junocam supports a technique called Time Delay Integration (TDI). To EFB03, TDI 60 has been applied, meaning the imagage was digitally shifted by 60 - 1 pixels during one exposure. This needs to be considered when applying EFB03 analysis results to other Junocam images.

The current article is intended to provide methods for a refinement of the laboratory results suggsting, that "there was little evidence of structure in the leakage" ([1, subsection 4.8]).

Raw images are provided square root encoded. Linear data are obtained by squaring the raw color values.

Let (x_1, y_1) and $(x_2, y_2) = (ax_1, ay_1)$ be two pairs of non-zero real numbers with a a real number. Then $x_2/x_1 = a = y_2/y_1$. Section 2 generalizes this simple test of linear dependence to a test of linear dependence of a set of framelets. Sections 3 and 4 summarize preliminary results of the tests applied to EFB03.

Most of this article requires some basic knowledge of linear algebra.

2 Stray Light Framelets as Weighted Sums

2.1 Linear Dependence of Stray Light Framelets

Let $\mathbf{N}$ denote the set of the natural numbers, $\mathbf{R}$ the field of the real numbers, and $\mathbf{R}_\mathbf{0}^+$ the subset of non-negative real numbers.

Let $2n$ stray light framelets j, and n template framelets i be all of the same size $r \times c$, with $r, c \in \mathbf{N}$. Let $n \in \mathbf{N}$ be a natural number. Assume each stray light framelet j being a weighted sum of the n template framelets i. Consider $2n$ valid relative pixel positions.

- Denote the linearized value at pixel position k within stray light framelet j with $v_{k,j}$.
- Denote the linearized value at pixel position k within template framelet i with $x_{k,i}$.
- For stray light framelet j denote the weight of framelet template i with $a_{i,j}$.

The linearized value $v_{k,j}$ at pixel position k within stray light framelet j can be written as weighted sum

$$v_{k,j} = \sum_{i=1}^{n} x_{k,i} a_{i,j} \tag{1}$$

of the corresponding linearized values $x_{k,i}$ at pixel position k within each template framelet i, weighted by $a_{i,j}$.

This system of equations can be summarized within one matrix equation

$$\begin{pmatrix} v_{1,1} & \cdots & v_{1,n} & v_{1,n+1} & \cdots & v_{1,2n} \\ \vdots & & \vdots & \vdots & & \vdots \\ v_{n,1} & \cdots & v_{n,n} & v_{n,n+1} & \cdots & v_{n,2n} \\ v_{n+1,1} & \cdots & v_{n+1,n} & v_{n+1,n+1} & \cdots & v_{n+1,2n} \\ \vdots & & \vdots & \vdots & & \vdots \\ v_{2n,1} & \cdots & v_{2n,n} & v_{2n,n+1} & \cdots & v_{2n,2n} \end{pmatrix} =$$

$$\begin{pmatrix} x_{1,1} & \cdots & x_{1,n} \\ \vdots & & \vdots \\ x_{n,1} & \cdots & x_{n,n} \\ x_{n+1,1} & \cdots & x_{n+1,n} \\ \vdots & & \vdots \\ x_{2n,1} & \cdots & x_{2n,n} \end{pmatrix} \cdot \begin{pmatrix} a_{1,1} & \cdots & a_{1,n} & a_{1,n+1} & \cdots & a_{1,2n} \\ \vdots & & \vdots & \vdots & & \vdots \\ a_{n,1} & \cdots & a_{n,n} & a_{n,n+1} & \cdots & a_{n,2n} \end{pmatrix},$$

or by defining block matrices

$$\begin{aligned} V_{1,1} &:= \begin{pmatrix} v_{1,1} & \cdots & v_{1,n} \\ \vdots & & \vdots \\ v_{n,1} & \cdots & v_{n,n} \end{pmatrix}, \\ V_{1,2} &:= \begin{pmatrix} v_{1,n+1} & \cdots & v_{1,2n} \\ \vdots & & \vdots \\ v_{n,n+1} & \cdots & v_{n,2n} \end{pmatrix}, \\ V_{2,1} &:= \begin{pmatrix} v_{n+1,1} & \cdots & v_{n+1,n} \\ \vdots & & \vdots \\ v_{2n,1} & \cdots & v_{2n,n} \end{pmatrix}, \end{aligned}$$

$$V_{2,1} := \begin{pmatrix} v_{n+1,n+1} & \cdots & v_{n+1,2n} \\ \vdots & & \vdots \\ v_{2n,n+1} & \cdots & v_{2n,2n} \end{pmatrix},$$

$$X_1 := \begin{pmatrix} x_{1,1} & \cdots & x_{1,n} \\ \vdots & & \vdots \\ x_{n,1} & \cdots & x_{n,n} \end{pmatrix},$$

$$X_2 := \begin{pmatrix} x_{n+1,1} & \cdots & x_{n+1,n} \\ \vdots & & \vdots \\ x_{2n,1} & \cdots & x_{2n,n} \end{pmatrix},$$

$$A_1 := \begin{pmatrix} a_{1,1} & \cdots & a_{1,n} \\ \vdots & & \vdots \\ a_{n,1} & \cdots & a_{n,n} \end{pmatrix}, \text{ and}$$

$$A_2 := \begin{pmatrix} a_{1,n+1} & \cdots & a_{1,2n} \\ \vdots & & \vdots \\ a_{n,n+1} & \cdots & a_{n,2n} \end{pmatrix},$$

written in block form

$$\begin{pmatrix} V_{1,1} & V_{1,2} \\ V_{2,1} & V_{2,2} \end{pmatrix} = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \cdot \begin{pmatrix} A_1 & A_2 \end{pmatrix}. \tag{2}$$

This block form can be decomposed into a system of four individual matrix equations:

$$V_{1,1} = X_1 A_1, \tag{3}$$

$$V_{1,2} = X_1 A_2, \tag{4}$$

$$V_{2,1} = X_2 A_1, \tag{5}$$

$$V_{2,2} = X_2 A_2. \tag{6}$$

Assume all $V_{l,m}$ and A_m as regular. By multiplying the inverse of the respective A_m from the right we get

$$X_1 = V_{1,1} A_1^{-1}, \tag{7}$$

$$X_1 = V_{1,2} A_2^{-1}, \tag{8}$$

$$X_2 = V_{2,1} A_1^{-1}, \tag{9}$$

$$X_2 = V_{2,2} A_2^{-1}. \tag{10}$$

Equations (7) and (8) say

$$V_{1,2} A_2^{-1} = X_1 = V_{1,1} A_1^{-1}, \tag{11}$$

hence by multiplying $V_{1,1}^{-1}$ from the left

$$A_1^{-1} = V_{1,1}^{-1} V_{1,2} A_2^{-1}. \tag{12}$$

Equations (9) and (10) say

$$V_{2,2} A_2^{-1} = X_2 = V_{2,1} A_1^{-1}, \tag{13}$$

hence by multiplying $V_{2,1}^{-1}$ from the left

$$A_1^{-1} = V_{2,1}^{-1} V_{2,2} A_2^{-1}. \tag{14}$$

Equations (12) and (14) written as one equation

$$V_{2,1}^{-1} V_{2,2} A_2^{-1} = A_1^{-1} = V_{1,1}^{-1} V_{1,2} A_2^{-1}, \tag{15}$$

and multiplication of A_2 from the right simplifies to

$$V_{2,1}^{-1} V_{2,2} = V_{1,1}^{-1} V_{1,2}. \tag{16}$$

This equation can be resolved, e.g. to $V_{2,2}$ by multiplying $V_{2,1}$ from the left:

$$V_{2,2} = V_{2,1} V_{1,1}^{-1} V_{1,2}. \tag{17}$$

The preceding calculations can be summarized as a

Lemma 1 *Let $n \in \mathbf{N}$. Then the equation system*

$$\forall 1 \le k, j \le 2n : v_{k,j} = \sum_{i=1}^{n} x_{k,i} a_{i,j}, \tag{18}$$

all $v_{k,j}$, $x_{k,i}$, $a_{i,j}$ elements of a field F, can be written as a system

$$V_{1,1} = X_1 A_1, \;\; V_{1,2} = X_1 A_2, \;\; V_{2,1} = X_2 A_1, \;\; V_{2,2} = X_2 A_2 \tag{19}$$

of four matrix equations, all matrices $n \times n$.

If A_1, A_2, $V_{1,1}$, and $V_{2,1}$ regular, then

$$V_{2,2} = V_{2,1} V_{1,1}^{-1} V_{1,2},$$

and

$$A_2 = A_1 V_{1,1}^{-1} V_{1,2} = A_1 V_{2,1}^{-1} V_{2,2}. \tag{20}$$

Equation (20) remains to be shown. It's obtained by multiplying A_1 from the left and A_2 from the right to equations (12) and (14). $\triangle$

Equation (16) is more convenient for test purposes than equation (17), hence

Corollary 2 *Let $n \in \mathbf{N}$. Write the equation system*

$$\forall 1 \leq k, j \leq 2n : v_{k,j} = \sum_{i=1}^{n} x_{k,i} a_{i,j},$$

all $v_{k,j}$, $x_{k,i}$, $a_{i,j}$ elements of a field F, as a system

$$V_{1,1} = X_1 A_1, \;\; V_{1,2} = X_1 A_2, \;\; V_{2,1} = X_2 A_1, \;\; V_{2,2} = X_2 A_2$$

of four matrix equations, all matrices $n \times n$.

If A_1, A_2, $V_{1,1}$, and $V_{2,1}$ regular, then

$$V_{2,1}^{-1} V_{2,2} = V_{1,1}^{-1} V_{1,2}.$$

$\triangle$

2.2 Testing Stray Light Framelets for Linear Dependence

Corollary 2 provides a means to test stray light framelets, e.g. those of EFB03, for linear dependence. In paractice, however, data tend to be noisy, and a test over all possible combinations of pixel positions and framelets isn't feasible for today's computer hardware. Appropriate statistical methods of the Monte Carlo family can account for both issues.

Randomly choosing a set of $2n$ pairwise different framelets, a set of $2n$ pairwise different relative pixel positions, and applying corollary 2 to the grey values at the relative pixel positions for the respective framelet positions returns the two matrices $S := V_{2,1}^{-1} V_{2,2}$ and $T := V_{1,1}^{-1} V_{1,2}$ of the corollary. For each of the n^2 components $s_{k,j}$ and $t_{k,j}$ of each these two matrices

$$S = \begin{pmatrix} s_{1,1} & \cdots & s_{1,n} \\ \vdots & & \vdots \\ s_{n,1} & \cdots & s_{n,n} \end{pmatrix}$$

and

$$T = \begin{pmatrix} t_{1,1} & \cdots & t_{1,n} \\ \vdots & & \vdots \\ t_{n,1} & \cdots & t_{n,n} \end{pmatrix}$$

the pair $(s_{k,j}, t_{k,j})$ can be written as a dot in a respective 2-dimensional coordinate sytem at position $(s_{k,j}, t_{k,j})$. Repeating this for a sufficiently large number of samples visualizes the correlation of S and T. The result is a $n \times n$-matrix of correlation diagrams.

3 Preliminary Results

Performing the test over all framelets and pixel positions $24 \leq x \leq 1630$, $0 \leq y \leq 87$ within the framelets for $1 \leq n \leq 6$ with a correction of $\gamma = 2.0$, and $1.6 \cdot 10^5$ samples each test run, did show some, but not a good correlation of the matrices S and T for $n \leq 3$. For $n \geq 5$ the correlation is barely visible.

The results are preliminary, since the applied software needs further tests. Correlation coefficients haven't been calculated. Determining these coefficients might be considered useful in future work.

4 Preliminary Conclusions

The correlation for small n suggests the framelets being a linear combination of up to about 3 template framelets, but with a significant contribution of either noise or structure induced by the structure of the target object, or both. A major contribution of structure induced by the structure of the target object appears plausible.

The conclusions are preliminary, since besides further software tests, cross checking the presumed contribution of the structure of the target object would be desirable.

Tesing linear dependence of selected framelets or pixel ranges might add further insight.

References

[1] C.J. Hansen, M.A. Caplinger, A. Ingersoll, M.A. Ravine, E. Jensen, S. Bolton, G. Orton. *Junocam: Juno's Outreach Camera.* Space Sci Rev DOI 10.1007/s11214-014-0079-x, Springer, 2014