Some Empirical Approximations of the Stray Light in Junocam Image EFB03, Part I

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Abstract

The image EFB03 taken by Junocam during Earth flyby shows mostly stray light. Therefore EFB03 is particularly well-suited to analyse the structure of stray light in Junocam images. The structure of the stray light in EFB03 can be approximated by empirical formulas. This article investigates approximations by sets of 1-dimensional linear functions and sets of 1-dimensional Gauss functions. [1]

[1]This document was typeset with LaTeX.

This article has been published as part of a post at `unmannedspaceflight.com`.

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1 Introduction

Images taken by Juno's Education and Outreach camera Junocam during the Earth flyby (EFB) in October 2013 are providing a first publicly available set of in-flight tests, similar to the images expected to be taken during Juno's Jupiter mission starting in mid-2016. So this article may be put into the context of [1, subsection 6.4], goal 3: "Provide data to the amateur image processing community and encourage them to produce a variety of products". Raw images are provided online by Malin Space Science Systems, San Diego, CA, USA (MSSS) [2].

Laboratory results suggested, that "there was little evidence of structure in the leakage" ([1, subsection 4.8]).

Deciphering this subtle structure may allow for a more accurate calibration of Junocam, and hence better quality of the calibrated images and higher science value.

Junocam returned 17 images during Earth flyby, named EFB01 to EFB17. The image EFB03 taken by Junocam during Earth flyby shows significant stray light. It doesn't show many primary objects. Therefore EFB03 is particularly well-suited to analyse the structure of stray light in Junocam images.

EFB03 consists of framelets of only one (the green) color channel. Each framelet consists of 128×1648 pixels.

The grey values of consecutive framelets don't behave completely random, but appear to change gradually. Grey values within each framelet appear to change gradually as well, for most of the area. This gradual change can be approximated by empirical formulas.

This article intends to find some of those formulas.

To avoid motion blur due to the rotation of the Juno probe, Junocam supports a technique called Time Delay Integration (TDI). For EFB03, TDI 60 has been applied, meaning the image was digitally shifted by 60 - 1 pixels during one exposure.

When looking at an individual framelet, two horizontal stripes can be perceived. Numerically averaging each of the 128 horizontal pixel rows of each framelet within some chosen horizontal interval reveals three distinguished horizontal substripes per framelet, after drawing the averaged values as a plot of a function of the vertical pixel position. The function can be approximated by a piece-wise linear function, one linear function per substripe. Section 2 treats this approximation in more detail.

The decomposition into substripes appears to be induced by the TDI technique.

A linear function can be described by two parameters. These parameters change over the framelets for according substripes. One of the two parameters, expressed by the value of the linear function at the center of a substripe, behaves similar to Gauss functions. Section 3 elaborates an according approximation.

Raw images are provided square root encoded. Linear data are obtained by squaring the raw color values.

2 Approximation by Piecewise Linear Functions

2.1 Regression Lines

EFB03 consists of 82 framelets of width 1648 and height 128 pixels. Let

$$v : \langle 0; 82 \cdot 128 - 1\rangle \times \langle 0; 1648 - 1\rangle \to \mathbf{R} \tag{1}$$

describe the linearized grey values of EFB03 as a map from a cartesian product of the intervals $\langle 0; 82 \cdot 128 - 1\rangle$ and $\langle 0; 1648 - 1\rangle$ of integers to the real numbers $\mathbf{R}$. Let x_1, x_2 be integer numbers with $0 \leq x_1 < x_2 < 1648$. Plotting the function

$$\begin{aligned} w_{x_1,x_2} : \langle 0; 82 \cdot 128 - 1\rangle &\to \mathbf{R}, && (2)\\ y &\mapsto \frac{1}{x_2 - x_1} \sum_{x_1}^{x_2-1} v(y, x) && (3) \end{aligned}$$

of horizontal averages over a given range $\langle x_1; x_2 - 1\rangle$ reveals by visual inspection, that w_{x_1,x_2} can be approximated by three linear functions

$$\begin{aligned} f_{x_1,x_2,j,s} : \langle 0; 82 \cdot 128 - 1\rangle &\to \mathbf{R}, && (4)\\ y &\mapsto \alpha_{x_1,x_2,j,s,1} x + \alpha_{x_1,x_2,j,s,0} && (5) \end{aligned}$$

(of type $ax + b$) for each framelet j, defined on disjoint domains s. These domains induce substripes by their cartesian product with an horizontal range. Below substripes are referenced by the same number s as the according domains.

Some try and error suggest

- domain 0 as the offset interval $[0; 83)$,
- domain 1 as $[83; 90)$, and
- domain 2 as $[90; 128)$,

relative to the y-coordinate $128j$ for each framelet j being a good initial choice.

With the bounds

$$\begin{aligned} y_{j,0,1} &:= 128j, && (6)\\ y_{j,0,2} = y_{j,1,1} &:= 128j + 83, && (7)\\ y_{j,1,2} = y_{j,2,1} &:= 128j + 90, \text{ and} && (8)\\ y_{j,2,2} &:= 128j + 128 && (9) \end{aligned}$$

of these domains, the parameters $\alpha_{x_1,x_2,j,s,0}$ and $\alpha_{x_1,x_2,j,s,1}$ can be determined the usual way parameters of a regression line through a set of points in a 2-dimensional coordinate system are determined, e.g. by

$$\alpha_{x_1,x_2,j,s,1} = \frac{\sum_{y=y_{j,s,1}}^{y_{j,s,2}-1} (y - \bar{y}_{j,s})(w_{x_1,x_2}(y) - \bar{w}_{j,s})}{\sum_{y=y_{j,s,1}}^{y_{j,s,2}-1} (y - \bar{y}_{j,s})^2} \tag{10}$$

$$\alpha_{x_1,x_2,j,s,0} = \bar{w}_{j,s} - \alpha_{x_1,x_2,j,s,1} \cdot \bar{y}_{j,s} \ , \tag{11}$$

with the mean y value

$$\bar{y}_{j,s} := \frac{\sum_{y=y_{j,s,1}}^{y_{j,s,2}-1} y}{y_{j,s,2} - y_{j,s,1}} = \frac{y_{j,s,1} + y_{j,s,2} - 1}{2} \tag{12}$$

and the mean $w(y)$ value

$$\bar{w}_{j,s} := \frac{\sum_{y=y_{j,s,1}}^{y_{j,s,2}-1} w_{x_1,x_2}(y)}{y_{j,s,2} - y_{j,s,1}} \tag{13}$$

for substripe s of framelet j.

Appendix A.1 lists calculated regression lines for $x_1 = 24$, and $x_2 = 1200$ of a linearized version of EFB03.

2.2 Selected One-Dimensional Time Series

2.2.1 Approach with Bias and Ascent

Considering the mean brightness

$$\begin{aligned} F_{x_1,x_2,s} : \langle 0; 81\rangle &\rightarrow \mathbf{R}, \\ j &\mapsto f_{x_1,x_2,j,s}\left(128j + \frac{y_{j,s,1} + y_{j,s,2} - 1}{2}\right) \end{aligned}$$

for each of the three respective substripes s separately along all framelets j simplifies further investigation.

A similar approach applies to the vertical change of brightness within each substripe:

$$\begin{aligned} A_{x_1,x_2,s} : \langle 0; 81\rangle &\rightarrow \mathbf{R}, \\ j &\mapsto \alpha_{x_1,x_2,j,s}\left(128j + \frac{y_{j,s,1} + y_{j,s,2} - 1}{2}\right). \end{aligned}$$

2.2.2 Approach with Two Function Values

Considering the mean brightness

$$\begin{aligned} F_{x_1,x_2,s,1} : \langle 0; 81\rangle &\rightarrow \mathbf{R}, \\ j &\mapsto f_{x_1,x_2,j,s}\left(128j + \frac{y_{j,s,1}}{2}\right) \end{aligned}$$

$$\begin{aligned} F_{x_1,x_2,s,2} : \langle 0; 81\rangle &\rightarrow \mathbf{R}, \\ j &\mapsto f_{x_1,x_2,j,s}\left(128j + \frac{y_{j,s,2}}{2}\right) \end{aligned}$$

at the lower and upper bound of the substripes avoids the need to approximate the ascent directly.

Assuming

$$F_{x_1,x_2,1,1} = F_{x_1,x_2,0,2} \tag{14}$$

and

$$F_{x_1,x_2,1,2} = F_{x_1,x_2,2,1} \tag{15}$$

describes substripe 1 by substripes 0 and 2. This constraint appears to be approximately valid for EFB03.

3 Approximation by Gauss Functions

3.1 Estimating the Parameters

This subsection isn't specific to Junocam. The convention for variables, particularly x and y aren't Junocam image coordinates.

3.1.1 Basic Properties of the Gauss Functions

Lemma 1 *Let $x, y, \mu \in \mathbf{R}$ be real numbers with $y > 1$, and $x > \mu$. Let*

$$\sigma = \frac{x-\mu}{\sqrt{2\ln y}}. \tag{16}$$

Then

$$e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} = \frac{1}{y}, \tag{17}$$

with $\sigma > 0$.

Since $y > 1$ by assumption, $\ln y$ is defined in $\mathbf{R}$, and $\ln y > 0$. Hence $\sqrt{\ln y}$, and $\sqrt{2\ln y}$ are defined in $\mathbf{R}$. The square root is defined as the non-negative root, hence $\sqrt{2\ln y} > 0$ With $x > \mu$ by assumption, $x - \mu > 0$. Hence $\sigma = \frac{x-\mu}{\sqrt{2\ln y}} > 0$. Applying equation (16) to the left side of equation (17), and straightforward calculation show equation (17)

$$\begin{aligned}
e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} &= e^{-\frac{1}{2}\cdot\left(\frac{x-\mu}{\frac{x-\mu}{\sqrt{2\ln y}}}\right)^2} && (18)\\
&= e^{-\frac{1}{2}\left(\sqrt{2\ln y}\right)^2} && (19)\\
&= e^{-\frac{1}{2}(2\ln y)} && (20)\\
&= e^{-\ln y} && (21)\\
&= \frac{1}{e^{\ln y}} && (22)\\
&= \frac{1}{y}.\ \triangle && (23)
\end{aligned}$$

Lemma 2 *Let $\mu, \sigma, \xi \in \mathbf{R}$, and $\sigma > 0$. Then*

$$e^{-\frac{1}{2}\left(\frac{(\mu-\xi)-\mu}{\sigma}\right)^2} = e^{-\frac{1}{2}\left(\frac{(\mu+\xi)-\mu}{\sigma}\right)^2}. \tag{24}$$

Straightforward by the equality of the exponents:

$$\begin{aligned}
\left(\frac{(\mu-\xi)-\mu}{\sigma}\right)^2 &= \left(\frac{-\xi}{\sigma}\right)^2 && (25)\\
= \left(\frac{\xi}{\sigma}\right)^2 &= \left(\frac{(\mu+\xi)-\mu}{\sigma}\right)^2.\ \triangle && (26)
\end{aligned}$$

Since the first derivative

$$\frac{\partial e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}{\partial x} = e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \cdot \left(-\frac{x-\mu}{\sigma}\right) \cdot \frac{1}{-\sigma} \tag{27}$$

$$= e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \cdot \frac{x-\mu}{\sigma^2} \tag{28}$$

of

$$G_{\mu,\sigma} : \mathbf{R} \to [0;1], \tag{29}$$

$$x \mapsto e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \tag{30}$$

equals zero only for $x = \mu$, and due to the smoothness of $G_{\mu,\sigma}$, the function $G_{\mu,\sigma}$ equals a given value $\frac{1}{y}$ in two points, at most. Due to the symmetry by Lemma 2, if for $x_1 \neq x_2$, $G_{\mu,\sigma}(x_1) = G_{\mu,\sigma}(x_2)$, then $\mu = \frac{x_1+x_2}{2}$ by $x_1 = \mu - \xi$ and $x_2 = \mu + \xi$.

So we get the

Corollary 3 *Let $G_{\mu,\sigma}(x_1) = G_{\mu,\sigma}(x_2)$, and $x_1 \neq x_2$. Then*

$$\mu = \frac{x_1 + x_2}{2}.$$

△

Lemma 4 *For $\mu, \sigma \in \mathbf{R}$, $\sigma > 0$:*

$$G_{\mu,\sigma}(\mu) = 1. \tag{31}$$

For $x \in \mathbf{R}$:

$$G_{\mu,\sigma}(x) \leq 1. \tag{32}$$

Trivial:

$$G_{\mu,\sigma}(\mu) = e^{-\frac{1}{2}\left(\frac{\mu-\mu}{\sigma}\right)^2} = e^{-\frac{1}{2}\left(\frac{0}{\sigma}\right)^2} = e^0 = 1,$$

and $\left(\frac{x-\mu}{\sigma}\right)^2 \geq 0$, since it's a square, hence $z := -\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2 \leq 0$, and $G_{\mu,\sigma}(x) = e^z \leq 1$. △

3.1.2 An estimator

Let $F : \mathbf{R} \mapsto \mathbf{R}^+$ be a continuous function with $F(x) > 0$ for any $x \in \mathbf{R}$, with

$$a := \max_{x \in \mathbf{R}} F(x) < \infty. \tag{33}$$

Let $0 < y \in \mathbf{R}$, and $F(x_1) = F(x_2) = \frac{a}{y}$, with $x_1 < x_2$, and $F(x) \neq \frac{a}{y}$ for $x_1 \neq x \neq x_2$. Assume, that there exists an $0 < \epsilon < x_2 - x_1$, with $F(x_1 - \epsilon) < \frac{a}{y}$, $F(x_2 + \epsilon) < \frac{a}{y}$, $F(x_1 + \epsilon) > \frac{a}{y}$, and (redundantly) $F(x_2 - \epsilon) > \frac{a}{y}$.

Let

$$\mu := \frac{x_1 + x_2}{2} \tag{34}$$

and

$$\sigma := \frac{x_2 - x_1}{2\sqrt{2 \ln y}}. \tag{35}$$

Then the function $H := \frac{1}{a}F$ shares the following properties with $G_{\mu,\sigma}$:

1. $\forall x \in \mathbf{R} : 0 < H(x) \leq 1$,
2. $\max H = \max G_{\mu,\sigma} = 1$,
3. $H(x_1) = G_{\mu,\sigma}(x_1)$, and $H(x_2) = G_{\mu,\sigma}(x_2)$.

Equation (34) holds according to corollary 3.

Using this equation for μ, equation (16) can be transformed to the same form as equation (35) by

$$\sigma = \frac{x_2 - \mu}{\sqrt{2 \ln y}} = \frac{x_2 - \frac{x_1+x_2}{2}}{\sqrt{2 \ln y}} = \frac{\frac{2x_2-x_1-x_2}{2}}{\sqrt{2 \ln y}} = \frac{\frac{x_2-x_1}{2}}{\sqrt{2 \ln y}} = \frac{x_2 - x_1}{2\sqrt{2 \ln y}}.$$

For F of the form $a \cdot G_{\mu,\sigma}$, the parameters μ and σ are independent of the choice of y within the above constraints. For F of other type, however, the parameters μ and σ obtained by equations (34) and (35) are non-constant functions $\mu(y)$ and $\sigma(y)$ of y.

Averaging $\mu(y)$ and $\sigma(y)$ over an interval $[y_1; y_2]$ of y-values yields estimates for μ and σ of a mean $G_{\mu_{[y_1;y_2]},\sigma_{[y_1;y_2]}}$:

$$\mu_{[y_1;y_2]} = \frac{1}{y_2 - y_1} \int_{y_1}^{y_2} \mu(y) dy \tag{36}$$

and

$$\sigma_{[y_1;y_2]} = \frac{1}{y_2 - y_1} \int_{y_1}^{y_2} \sigma(y) dy. \tag{37}$$

For numerical purposes the average over a finite set $\{y_1, \ldots, y_n\}$ for some $n \in \mathbf{N}$ of y-values may be more suitable:

$$\mu_{\{y_1,\ldots,y_n\}} = \frac{1}{n} \sum_{y_1}^{y_n} \mu(y_n) \tag{38}$$

and

$$\sigma_{\{y_1,\ldots,y_n\}} = \frac{1}{n} \sum_{y_1}^{y_n} \sigma(y_n). \tag{39}$$

Calculating weighted averages using a Gauss blur would possibly introduce effects caused by the strong peak at the right of EFB03. Therefore it isn't used here.

3.2 Application to EFB03

3.2.1 Applications

Plotting $\mu(y)$ and $\sigma(y)$ as a function of y gives an idea of the presence of higher momenta, e.g. the skewedness, of the stray light along the subframes.

The piece-wise linear functions $F_{x_1,x_2,s}$ depend on the choice of the horizontal bounds x_1 and x_2. Applying some fixed of the suggested estimators of the Gauss function for a sequence of vertical stripes $[x_{m,1}, x_{m,2})$ of e.g. the same constant width $x_{m,2} - x_{m,1}$ for each vertical stripe m, and vertical stripes chosen equidistant, i.e. $x_{m+1,1} - x_{m,1}$ constant for each m, reveals the dependence of the approximated Gauss parameters from the horizontal position within EFB03. This can then be used to approximate the stray light of EFB03 in two dimensions instead of just one (vertical) dimension.

Subtracting an approximation of the stray light from the raw data enhances details of the thus far unconsidered residuals.

3.2.2 Limitations

The functions $F_{x_1,x_2,s}$ obtained from EFB03 are rather smooth for sufficiently large widths $x_2 - x_1$ of the vertical stripes, and for stripes not too far to the right, meaning about left of horizontal pixel position 1200. The above estimator is limited to functions of such benign properties.

A modification towards a map with domain $[0, 2\pi)$ instead of $\mathbf{R}$ might turn out to be more appropriate, since vertical coordinates correspond to rotation angles of the camera.

The functions $A_{x_1,x_2,s}$ appear less similar to Gauss functions, in general. They need to be approximated differently to obtain useful results.

3.2.3 Mitigation

Explicite estimation of $A_{x_1,x_2,s}$ can be avoided by approximating the lower bound $F_{x_1,x_2,s,1}$ and upper bound $F_{x_1,x_2,s,2}$ of the substripes instead of the average $F_{x_1,x_2,s}$ by Gauss functions. The according parameters of the approximating Gauss function are denoted by $a_{x_1,x_2,s,1}$, $\mu_{x_1,x_2,s,1}$, and $\sigma_{x_1,x_2,s,1}$ for the lower bound, and by $a_{x_1,x_2,s,2}$, $\mu_{x_1,x_2,s,2}$, $\sigma_{x_1,x_2,s,2}$ for the upper bound of substripe s.

Appendix B lists parameters of Gauss functions obtained from EFB03 with this method.

4 Future Refinements

The Gauss functions don't consider the rotation of the camera. Some modification of the Gauss functions may allow to account for this. The parameters of the - then possibly modified - Gauss functions are dependent of the horizontal position of the respective vertical stripes. It may be possible to find a simple formula which approximates those parameters reasonably well. These items are intended to be investigated in part II.

A Tables of Piecewise Linear Functions

The following subsections contain tables of regression lines for averages of linearized grey values of EFB03 over specific horizontal ranges.

A.1 Horizontal Average from Pixel 24 to 1200

The following tables describe regression lines for the averages of horizontal pixel position x, with $24 \leq x < 1200$.

Regression lines $f : y \mapsto \alpha_1 y + \alpha_0$ for averages over $24 \leq x < 1200$

Framelet j	Substripe s	y_1	y_2	α_0	α_1	$f(y_1)$	$f(y_2-1)$
0	0	0	82	0.082377	−0.000181	0.082377	0.067726
0	1	82	89	0.202560	−0.001619	0.069784	0.060068
0	2	89	128	0.102531	−0.000509	0.057220	0.037874
1	0	128	210	0.154214	−0.000212	0.127063	0.109881
1	1	210	217	0.617530	−0.002415	0.110327	0.095835
1	2	217	256	0.238179	−0.000685	0.089587	0.063567
2	0	256	338	0.340946	−0.000324	0.258022	0.231785
2	1	338	345	2.069819	−0.005425	0.236120	0.203569
2	2	345	384	0.863095	−0.001942	0.193205	0.119420
3	0	384	466	0.518381	−0.000351	0.383623	0.355197
3	1	466	473	4.622471	−0.009142	0.362359	0.307507
3	2	473	512	1.644651	−0.002861	0.291496	0.182786
4	0	512	594	0.790765	−0.000469	0.550666	0.512682
4	1	594	601	9.203868	−0.014607	0.527361	0.439720
4	2	601	640	2.929567	−0.004167	0.425108	0.266756
5	0	640	722	1.238076	−0.000623	0.839092	0.788595
5	1	722	729	13.525327	−0.017634	0.793265	0.687458
5	2	729	768	5.344646	−0.006443	0.647883	0.403059
6	0	768	850	1.568652	−0.000650	1.069293	1.016626
6	1	850	857	22.633738	−0.025407	1.037933	0.885492
6	2	857	896	8.060098	−0.008426	0.839358	0.519185
7	0	896	978	1.862674	−0.000559	1.361703	1.316414
7	1	978	985	32.922526	−0.032290	1.343198	1.149459
7	2	985	1024	11.521671	−0.010579	1.101298	0.699293
8	0	1024	1106	2.393480	−0.000645	1.733264	1.681040
8	1	1106	1113	45.164343	−0.039282	1.718953	1.483264
8	2	1113	1152	16.196766	−0.013274	1.422351	0.917924
9	0	1152	1234	3.269668	−0.001046	2.064568	1.979834
9	1	1234	1241	53.099804	−0.041406	2.004400	1.755962
9	2	1241	1280	20.592163	−0.015231	1.690990	1.112227

Regression lines $f : y \mapsto \alpha_1 y + \alpha_0$ for averages over $24 \leq x < 1200$

Framelet j	Substripe s	y_1	y_2	α_0	α_1	$f(y_1)$	$f(y_2 - 1)$
10	0	1280	1362	4.062859	−0.001297	2.403267	2.298246
10	1	1362	1369	64.482619	−0.045624	2.342780	2.069037
10	2	1369	1408	25.297153	−0.017024	1.990879	1.343956
11	0	1408	1490	4.605614	−0.001290	2.789478	2.684998
11	1	1490	1497	72.651894	−0.046940	2.711521	2.429882
11	2	1497	1536	29.836497	−0.018369	2.338849	1.640846
12	0	1536	1618	5.332141	−0.001421	3.148852	3.033718
12	1	1618	1625	79.029924	−0.046954	3.057935	2.776209
12	2	1625	1664	36.374114	−0.020716	2.709809	1.922582
13	0	1664	1746	6.638771	−0.001867	3.531277	3.380011
13	1	1746	1753	98.191749	−0.054289	3.403818	3.078087
13	2	1753	1792	39.065463	−0.020562	3.019409	2.238034
14	0	1792	1874	8.440107	−0.002421	4.102330	3.906259
14	1	1874	1881	127.560144	−0.065962	3.948287	3.552518
14	2	1881	1920	43.147094	−0.021097	3.462771	2.661068
15	0	1920	2002	10.204891	−0.002564	5.282736	5.075082
15	1	2002	2009	173.595731	−0.084140	5.148236	4.643398
15	2	2009	2048	55.810539	−0.025535	4.510793	3.540464
16	0	2048	2130	9.518704	−0.001101	7.263000	7.173785
16	1	2130	2137	251.855327	−0.114842	7.241849	6.552797
16	2	2137	2176	65.361419	−0.027596	6.389833	5.341204
17	0	2176	2258	7.085532	0.000997	9.253934	9.334651
17	1	2258	2265	245.756908	−0.104688	9.371255	8.743127
17	2	2265	2304	58.360375	−0.021960	8.620850	7.786368
18	0	2304	2386	1.855849	0.004064	11.219459	11.548649
18	1	2386	2393	350.728996	−0.142136	11.591881	10.739064
18	2	2393	2432	52.586252	−0.017564	10.556387	9.888968
19	0	2432	2514	−5.348040	0.007669	13.302480	13.923653
19	1	2514	2521	487.241755	−0.188235	14.019722	12.890314
19	2	2521	2560	48.173817	−0.014098	12.631727	12.095987

Regression lines $f : y \mapsto \alpha_1 y + \alpha_0$ for averages over $24 \leq x < 1200$

Framelet j	Substripe s	y_1	y_2	α_0	α_1	$f(y_1)$	$f(y_2 - 1)$
20	0	2560	2642	−14.545509	0.011584	15.110502	16.048836
20	1	2642	2649	632.330712	−0.233205	16.202609	14.803378
20	2	2649	2688	37.454686	−0.008678	14.465607	14.135828
21	0	2688	2770	−26.019366	0.015764	16.353467	17.630327
21	1	2770	2777	726.895000	−0.256014	17.736680	16.200597
21	2	2777	2816	27.708358	−0.004282	15.817903	15.655196
22	0	2816	2898	−30.795333	0.016896	16.783454	18.152020
22	1	2898	2905	811.257745	−0.273622	18.301683	16.659952
22	2	2905	2944	18.876434	−0.000883	16.311165	16.277609
23	0	2944	3026	−32.504012	0.016547	16.210517	17.550829
23	1	3026	3033	820.009690	−0.265137	17.705106	16.114284
23	2	3033	3072	18.582893	−0.000939	15.735728	15.700056
24	0	3072	3154	−28.708995	0.014154	14.771275	15.917727
24	1	3154	3161	777.743366	−0.241500	16.051450	14.602448
24	2	3161	3200	30.595363	−0.005179	14.223515	14.026701
25	0	3200	3282	−17.206146	0.009350	12.714672	13.472043
25	1	3282	3289	651.652529	−0.194426	13.545275	12.378717
25	2	3289	3328	43.955335	−0.009694	12.071956	11.703586
26	0	3328	3410	−4.099831	0.004340	10.343500	10.695036
26	1	3410	3417	509.503936	−0.146260	10.758784	9.881227
26	2	3417	3456	54.706199	−0.013174	9.690612	9.190000
27	0	3456	3538	−6.447959	0.004077	7.643877	7.974154
27	1	3538	3545	502.198125	−0.139666	8.061554	7.223560
27	2	3545	3584	56.057148	−0.013836	7.008102	6.482329
28	0	3584	3666	0.091906	0.001389	5.070066	5.182575
28	1	3666	3673	349.672007	−0.093952	5.242282	4.678567
28	2	3673	3712	76.304522	−0.019540	4.535331	3.792824
29	0	3712	3794	4.597027	−0.000323	3.397818	3.371650
29	1	3794	3801	229.740430	−0.059657	3.402835	3.044895
29	2	3801	3840	60.071595	−0.015021	2.975375	2.404563

Regression lines $f : y \mapsto \alpha_1 y + \alpha_0$ for averages over $24 \le x < 1200$

Framelet j	Substripe s	y_1	y_2	α_0	α_1	$f(y_1)$	$f(y_2-1)$
30	0	3840	3922	3.242731	−0.000171	2.587328	2.573503
30	1	3922	3929	201.228491	−0.050645	2.597789	2.293917
30	2	3929	3968	67.755456	−0.016681	2.215259	1.581376
31	0	3968	4050	1.926886	0.000013	1.977961	1.979003
31	1	4050	4057	192.640370	−0.047069	2.010654	1.728240
31	2	4057	4096	58.123926	−0.013920	1.652228	1.123285
32	0	4096	4178	0.226572	0.000326	1.560827	1.587213
32	1	4178	4185	172.076089	−0.040802	1.606181	1.361371
32	2	4185	4224	50.104257	−0.011660	1.305668	0.862574
33	0	4224	4306	−0.168744	0.000311	1.145085	1.170279
33	1	4306	4313	136.615499	−0.031450	1.189916	1.001214
33	2	4313	4352	37.029645	−0.008367	0.943182	0.625239
34	0	4352	4434	0.101429	0.000154	0.769839	0.782280
34	1	4434	4441	88.412132	−0.019761	0.792388	0.673823
34	2	4441	4480	26.185289	−0.005754	0.632513	0.413867
35	0	4480	4562	−0.110854	0.000147	0.549864	0.561810
35	1	4562	4569	70.539165	−0.015336	0.575176	0.483159
35	2	4569	4608	17.903350	−0.003820	0.450595	0.305442
36	0	4608	4690	−0.101835	0.000091	0.318130	0.325512
36	1	4690	4697	42.269686	−0.008942	0.332686	0.279035
36	2	4697	4736	11.114755	−0.002311	0.260515	0.172701
37	0	4736	4818	0.135735	0.000011	0.189700	0.190623
37	1	4818	4825	22.958474	−0.004726	0.190052	0.161698
37	2	4825	4864	6.628241	−0.001342	0.152096	0.101092
38	0	4864	4946	0.437167	−0.000073	0.084491	0.078618
38	1	4946	4953	10.175579	−0.002041	0.078756	0.066508
38	2	4953	4992	2.113577	−0.000414	0.062662	0.046927
39	0	4992	5074	0.818320	−0.000159	0.022916	0.010010
39	1	5074	5081	1.668550	−0.000326	0.012193	0.010234
39	2	5081	5120	0.065818	−0.000011	0.009863	0.009445

Regression lines $f : y \mapsto \alpha_1 y + \alpha_0$ for averages over $24 \le x < 1200$

Framelet j	Substripe s	y_1	y_2	α_0	α_1	$f(y_1)$	$f(y_2-1)$
40	0	5120	5202	0.900577	−0.000172	0.017748	0.003781
40	1	5202	5209	0.518424	−0.000099	0.005450	0.004858
40	2	5209	5248	−0.061558	0.000013	0.005465	0.005954
41	0	5248	5330	0.909687	−0.000170	0.015854	0.002058
41	1	5330	5337	0.431683	−0.000080	0.003843	0.003362
41	2	5337	5376	−0.108034	0.000021	0.003817	0.004614
42	0	5376	5458	0.925454	−0.000169	0.014572	0.000848
42	1	5458	5465	0.535792	−0.000098	0.002774	0.002188
42	2	5465	5504	−0.192211	0.000036	0.002545	0.003900
43	0	5504	5586	0.941674	−0.000169	0.013966	0.000314
43	1	5586	5593	0.651557	−0.000116	0.002257	0.001560
43	2	5593	5632	−0.253710	0.000046	0.002055	0.003793
44	0	5632	5714	0.988397	−0.000173	0.014017	0.000004
44	1	5714	5721	0.574987	−0.000100	0.001997	0.001396
44	2	5721	5760	−0.200357	0.000035	0.002074	0.003419
45	0	5760	5842	0.864553	−0.000148	0.013360	0.001390
45	1	5842	5849	0.703146	−0.000120	0.001826	0.001106
45	2	5849	5888	−0.309344	0.000053	0.001638	0.003659
46	0	5888	5970	1.047894	−0.000176	0.013996	−0.000227
46	1	5970	5977	0.221287	−0.000037	0.001588	0.001367
46	2	5977	6016	−0.259292	0.000044	0.001748	0.003408
47	0	6016	6098	1.083794	−0.000178	0.014049	−0.000354
47	1	6098	6105	0.965346	−0.000158	0.002348	0.001400
47	2	6105	6144	−0.211043	0.000035	0.001939	0.003264
48	0	6144	6226	1.129803	−0.000182	0.014254	−0.000453
48	1	6226	6233	0.400713	−0.000064	0.001793	0.001408
48	2	6233	6272	−0.272117	0.000044	0.001770	0.003439
49	0	6272	6354	1.145776	−0.000180	0.014152	−0.000462
49	1	6354	6361	0.810500	−0.000127	0.002311	0.001548
49	2	6361	6400	−0.285848	0.000045	0.001903	0.003622

Regression lines $f : y \mapsto \alpha_1 y + \alpha_0$ for averages over $24 \leq x < 1200$

Framelet j	Substripe s	y_1	y_2	α_0	α_1	$f(y_1)$	$f(y_2 - 1)$
50	0	6400	6482	1.082390	−0.000167	0.013552	0.000024
50	1	6482	6489	1.794606	−0.000276	0.002851	0.001192
50	2	6489	6528	−0.236623	0.000037	0.001993	0.003391
51	0	6528	6610	1.156920	−0.000175	0.013932	−0.000250
51	1	6610	6617	1.185236	−0.000179	0.002045	0.000971
51	2	6617	6656	−0.300221	0.000046	0.001764	0.003498
52	0	6656	6738	1.161209	−0.000172	0.013849	−0.000114
52	1	6738	6745	0.642334	−0.000095	0.001968	0.001397
52	2	6745	6784	−0.119780	0.000018	0.002675	0.003365
53	0	6784	6866	1.178789	−0.000172	0.013737	−0.000174
53	1	6866	6873	0.949106	−0.000138	0.002200	0.001372
53	2	6873	6912	−0.226905	0.000033	0.002022	0.003288
54	0	6912	6994	1.170827	−0.000167	0.013535	−0.000027
54	1	6994	7001	0.820052	−0.000117	0.002093	0.001391
54	2	7001	7040	−0.309754	0.000044	0.001764	0.003455
55	0	7040	7122	1.191453	−0.000167	0.013525	−0.000028
55	1	7122	7129	0.295266	−0.000041	0.001790	0.001543
55	2	7129	7168	−0.293088	0.000041	0.001979	0.003552
56	0	7168	7250	1.222157	−0.000169	0.013524	−0.000134
56	1	7250	7257	0.988003	−0.000136	0.001954	0.001138
56	2	7257	7296	−0.368860	0.000051	0.001625	0.003565
57	0	7296	7378	1.236266	−0.000168	0.013549	−0.000026
57	1	7378	7385	0.685495	−0.000093	0.001878	0.001322
57	2	7385	7424	−0.330382	0.000045	0.001677	0.003386
58	0	7424	7506	1.302043	−0.000174	0.013886	−0.000169
58	1	7506	7513	0.972950	−0.000129	0.002143	0.001367
58	2	7513	7552	−0.263396	0.000035	0.001916	0.003258
59	0	7552	7634	1.324197	−0.000173	0.013969	−0.000084
59	1	7634	7641	0.447219	−0.000058	0.001724	0.001374
59	2	7641	7680	−0.268313	0.000035	0.001865	0.003208

Regression lines $f : y \mapsto \alpha_1 y + \alpha_0$ for averages over $24 \leq x < 1200$

Framelet j	Substripe s	y_1	y_2	α_0	α_1	$f(y_1)$	$f(y_2 - 1)$
60	0	7680	7762	1.322976	−0.000170	0.013689	−0.000120
60	1	7762	7769	0.729421	−0.000094	0.001905	0.001342
60	2	7769	7808	−0.369888	0.000048	0.001658	0.003475
61	0	7808	7890	1.378474	−0.000175	0.013872	−0.000284
61	1	7890	7897	0.422730	−0.000053	0.001761	0.001441
61	2	7897	7936	−0.342565	0.000044	0.001801	0.003458
62	0	7936	8018	1.356993	−0.000169	0.013665	−0.000046
62	1	8018	8025	0.745716	−0.000093	0.001844	0.001288
62	2	8025	8064	−0.327770	0.000041	0.001907	0.003468
63	0	8064	8146	1.335604	−0.000164	0.013360	0.000079
63	1	8146	8153	0.354860	−0.000043	0.001725	0.001465
63	2	8153	8192	−0.324274	0.000040	0.001850	0.003370
64	0	8192	8274	1.404965	−0.000170	0.013537	−0.000221
64	1	8274	8281	0.784351	−0.000095	0.001948	0.001380
64	2	8281	8320	−0.375038	0.000046	0.001826	0.003555
65	0	8320	8402	1.412912	−0.000168	0.013572	−0.000052
65	1	8402	8409	1.082700	−0.000129	0.002010	0.001239
65	2	8409	8448	−0.333758	0.000040	0.001862	0.003379
66	0	8448	8530	1.485003	−0.000174	0.013863	−0.000243
66	1	8530	8537	0.349893	−0.000041	0.001445	0.001200
66	2	8537	8576	−0.400630	0.000047	0.001657	0.003448
67	0	8576	8658	1.458784	−0.000169	0.013627	−0.000022
67	1	8658	8665	1.231291	−0.000142	0.002188	0.001336
67	2	8665	8704	−0.336589	0.000039	0.001926	0.003410
68	0	8704	8786	1.463105	−0.000167	0.013522	0.000032
68	1	8786	8793	0.539857	−0.000061	0.002022	0.001655
68	2	8793	8832	−0.267666	0.000031	0.001951	0.003117
69	0	8832	8914	1.515497	−0.000170	0.013642	−0.000132
69	1	8914	8921	0.295733	−0.000033	0.001666	0.001468
69	2	8921	8960	−0.319312	0.000036	0.001918	0.003287

Regression lines $f : y \mapsto \alpha_1 y + \alpha_0$ for averages over $24 \leq x < 1200$

Framelet j	Substripe s	y_1	y_2	α_0	α_1	$f(y_1)$	$f(y_2-1)$
70	0	8960	9042	1.505506	−0.000167	0.013548	0.000060
70	1	9042	9049	1.224321	−0.000135	0.002086	0.001275
70	2	9049	9088	−0.311575	0.000035	0.001766	0.003082
71	0	9088	9170	1.575770	−0.000172	0.013882	−0.000039
71	1	9170	9177	0.361995	−0.000039	0.001600	0.001365
71	2	9177	9216	−0.301996	0.000033	0.001976	0.003235
72	0	9216	9298	1.596091	−0.000172	0.013711	−0.000196
72	1	9298	9305	1.361936	−0.000146	0.002113	0.001236
72	2	9305	9344	−0.365962	0.000040	0.001764	0.003266
73	0	9344	9426	1.619741	−0.000172	0.013814	−0.000107
73	1	9426	9433	0.488889	−0.000052	0.001684	0.001374
73	2	9433	9472	−0.374198	0.000040	0.001916	0.003432
74	0	9472	9554	1.643376	−0.000172	0.013774	−0.000161
74	1	9554	9561	0.778906	−0.000081	0.001764	0.001276
74	2	9561	9600	−0.339942	0.000036	0.001849	0.003207
75	0	9600	9682	1.662254	−0.000172	0.013782	−0.000127
75	1	9682	9689	0.672904	−0.000069	0.001811	0.001395
75	2	9689	9728	−0.435674	0.000045	0.001846	0.003562
76	0	9728	9810	1.651299	−0.000168	0.013595	−0.000042
76	1	9810	9817	0.174513	−0.000018	0.001601	0.001495
76	2	9817	9856	−0.364839	0.000037	0.001881	0.003301
77	0	9856	9938	1.713048	−0.000172	0.013766	−0.000199
77	1	9938	9945	0.625482	−0.000063	0.001742	0.001365
77	2	9945	9984	−0.454890	0.000046	0.001669	0.003413
78	0	9984	10066	1.763060	−0.000175	0.013937	−0.000254
78	1	10066	10073	1.615902	−0.000160	0.002300	0.001338
78	2	10073	10112	−0.319599	0.000032	0.002043	0.003256
79	0	10112	10194	1.824628	−0.000179	0.014415	−0.000086
79	1	10194	10201	1.128729	−0.000111	0.002084	0.001421
79	2	10201	10240	−0.327094	0.000032	0.002133	0.003360

Regression lines $f : y \mapsto \alpha_1 y + \alpha_0$ for averages over $24 \leq x < 1200$

Framelet j	Substripe s	y_1	y_2	α_0	α_1	$f(y_1)$	$f(y_2-1)$
80	0	10240	10322	1.803729	−0.000173	0.031935	0.017920
80	1	10322	10329	3.168013	−0.000305	0.019772	0.017942
80	2	10329	10368	0.720500	−0.000068	0.016564	0.013974
81	0	10368	10450	2.226187	−0.000206	0.090383	0.073697
81	1	10450	10457	20.823144	−0.001985	0.077816	0.065905
81	2	10457	10496	5.449897	−0.000515	0.062197	0.042618

B Tables of Approximations by Gauss Functions

The tables of this section list parameters for Gauss functions calculated from a linearized version of EFB03. Allowed grey values after linearization are between 0 and 255. Each Gauss function approximates a vertical stripe of width 100 pixels. Consecutive stripes are displaced horizontally by 50 pixels. Since the left 24 pixels of EFB03 are black, the first stripe starts at pixel position 24.

The parameters refer to horizontal substripes 0 and 2 with respect to each framelet. Bounds of the substripes have been assumed to be at relative vertical positions 0, 83, 90, and 128, the lower bound inclusive, the upper bound exclusive.

Convention for pixel positions is $(0, 0)$ for the lower left corner of EFB03.

B.1 Substripe 0

Approximations $a_{x_1,x_2,0,1}G(\mu_{x_1,x_2,0,1},\sigma_{x_1,x_2,0,1})$ of $F_{x_1,x_2,0,1}$
and $a_{x_1,x_2,0,2}G(\mu_{x_1,x_2,0,2},\sigma_{x_1,x_2,0,2})$ of $F_{x_1,x_2,0,2}$ by Gauss functions

x_1	x_2	$a_{x_1,x_2,0,1}$	$\mu_{x_1,x_2,0,1}$	$\sigma_{x_1,x_2,0,1}$	$a_{x_1,x_2,0,2}$	$\mu_{x_1,x_2,0,2}$	$\sigma_{x_1,x_2,0,2}$
24	124	18.012032	20.803717	3.937196	18.522949	21.369310	3.857250
74	174	16.736149	20.984796	4.066606	17.258174	21.438157	3.930064
124	224	15.302618	21.119780	4.157163	15.833248	21.495926	4.010495
174	274	13.981488	21.227522	4.313367	14.665566	21.527415	4.169457
224	324	13.154696	21.283574	4.429365	14.004696	21.523294	4.329534
274	374	12.978897	21.290817	4.521651	13.911148	21.510270	4.422387
324	424	13.218515	21.285711	4.593794	14.283032	21.495290	4.480765
374	474	13.587911	21.292598	4.600656	14.643586	21.504662	4.482330
424	524	13.477408	21.337696	4.607227	14.324982	21.529712	4.443773
474	574	12.980298	21.424841	4.565009	13.577943	21.597304	4.428492
524	624	12.704641	21.535628	4.530603	13.195847	21.619522	4.449814
574	674	12.906374	21.602578	4.507645	13.445564	21.652710	4.449999
624	724	13.423945	21.643810	4.466723	14.078265	21.683934	4.415378
674	774	14.077516	21.668463	4.429610	14.889062	21.694970	4.371102
724	824	14.848972	21.705736	4.393439	15.749921	21.711467	4.321916
774	874	15.700220	21.749565	4.356861	16.880769	21.745861	4.266251
824	924	16.554592	21.807232	4.314477	18.172061	21.764235	4.213685
874	974	17.595246	21.850486	4.268557	19.397391	21.806787	4.139505
924	1024	19.167153	21.914582	4.217922	21.360412	21.857223	4.019249
974	1074	21.820107	22.081207	4.105915	24.428187	21.941115	3.863155

Approximations $a_{x_1,x_2,0,1}G(\mu_{x_1,x_2,0,1},\sigma_{x_1,x_2,0,1})$ of $F_{x_1,x_2,0,1}$
and $a_{x_1,x_2,0,2}G(\mu_{x_1,x_2,0,2},\sigma_{x_1,x_2,0,2})$ of $F_{x_1,x_2,0,2}$ by Gauss functions

x_1	x_2	$a_{x_1,x_2,0,1}$	$\mu_{x_1,x_2,0,1}$	$\sigma_{x_1,x_2,0,1}$	$a_{x_1,x_2,0,2}$	$\mu_{x_1,x_2,0,2}$	$\sigma_{x_1,x_2,0,2}$
1024	1124	25.732450	22.125535	3.996286	28.281724	21.984038	3.725584
1074	1174	29.512560	22.165582	3.832075	32.088212	21.983388	3.676729
1124	1224	31.771919	22.256012	3.660504	34.207328	21.991162	3.642657
1174	1274	33.232583	22.264194	3.599269	35.078944	22.009196	3.598316
1224	1324	34.801074	22.220986	3.610004	36.251529	22.029738	3.642566
1274	1374	38.380201	22.284858	3.561594	39.850070	22.060169	3.568502
1324	1424	46.832039	22.390863	3.285413	49.207019	22.114408	3.249427
1374	1474	65.304820	22.770319	2.912633	68.971450	22.259000	2.874500
1424	1524	85.830342	22.952959	2.930060	94.143121	22.351086	2.900057
1474	1574	82.075037	22.838715	3.086240	90.991996	22.339283	3.013166
1524	1624	83.984837	22.723318	2.811497	82.585559	22.216996	2.913863

B.2 Substripe 2

Approximations $a_{x_1,x_2,2,1}G(\mu_{x_1,x_2,2,1}, \sigma_{x_1,x_2,2,1})$ of $F_{x_1,x_2,2,1}$
and $a_{x_1,x_2,2,2}G(\mu_{x_1,x_2,2,2}, \sigma_{x_1,x_2,2,2})$ of $F_{x_1,x_2,2,2}$ by Gauss functions

x_1	x_2	$a_{x_1,x_2,2,1}$	$\mu_{x_1,x_2,2,1}$	$\sigma_{x_1,x_2,2,1}$	$a_{x_1,x_2,2,2}$	$\mu_{x_1,x_2,2,2}$	$\sigma_{x_1,x_2,2,2}$
24	124	16.366029	21.390684	3.915310	19.102794	21.853796	3.507177
74	174	15.405113	21.464990	3.941407	18.016875	21.861557	3.552807
124	224	14.198107	21.495415	4.052478	16.295627	21.850420	3.637380
174	274	13.237066	21.525281	4.182882	14.675980	21.789354	3.775374
224	324	12.729635	21.526195	4.297804	14.116149	21.791238	3.875347
274	374	12.623034	21.514007	4.383267	14.469065	21.798924	3.906930
324	424	12.830524	21.504716	4.461090	14.747377	21.797471	3.961674
374	474	13.065851	21.502285	4.487693	14.661902	21.821627	4.011399
424	524	12.837436	21.533950	4.430875	14.227464	21.819894	4.001831
474	574	12.363589	21.589101	4.369026	13.267214	21.796831	4.005686
524	624	12.108023	21.622343	4.373114	12.425935	21.777881	4.083564
574	674	12.195063	21.632595	4.407892	12.111812	21.748449	4.167044
624	724	12.710425	21.651940	4.400747	12.133633	21.734515	4.257270
674	774	13.376158	21.687915	4.393609	12.538438	21.722438	4.269324
724	824	14.084874	21.708382	4.367935	13.129192	21.712330	4.229503
774	874	15.074804	21.733752	4.316105	13.861004	21.712463	4.220326
824	924	16.222312	21.740654	4.257962	14.658317	21.737941	4.198572
874	974	17.314721	21.737865	4.189855	15.551418	21.743366	4.131880
924	1024	18.938443	21.791930	4.073166	16.930185	21.760425	4.029563
974	1074	21.634377	21.858856	3.912593	19.050550	21.754579	3.891182

Approximations $a_{x_1,x_2,2,1}G(\mu_{x_1,x_2,2,1}, \sigma_{x_1,x_2,2,1})$ of $F_{x_1,x_2,2,1}$
and $a_{x_1,x_2,2,2}G(\mu_{x_1,x_2,2,2}, \sigma_{x_1,x_2,2,2})$ of $F_{x_1,x_2,2,2}$ by Gauss functions

x_1	x_2	$a_{x_1,x_2,2,1}$	$\mu_{x_1,x_2,2,1}$	$\sigma_{x_1,x_2,2,1}$	$a_{x_1,x_2,2,2}$	$\mu_{x_1,x_2,2,2}$	$\sigma_{x_1,x_2,2,2}$
1024	1124	25.022454	21.920469	3.745732	21.878863	21.754252	3.732370
1074	1174	28.703088	21.932157	3.644141	25.566070	21.758577	3.520456
1124	1224	31.346401	21.962661	3.534241	29.411073	21.772964	3.293406
1174	1274	32.573792	21.973831	3.467462	30.975884	21.792657	3.217026
1224	1324	33.720552	21.981495	3.480522	30.486734	21.827116	3.259265
1274	1374	36.628545	22.026617	3.442760	31.472417	21.862977	3.207650
1324	1424	44.329746	22.084543	3.217815	36.080118	21.889208	3.052112
1374	1474	60.051436	22.169641	2.956122	45.595380	21.899292	2.949807
1424	1524	81.726330	22.234044	2.970030	56.446338	21.900635	3.123565
1474	1574	79.558103	22.239255	3.070320	57.479419	21.910762	3.135096
1524	1624	72.684206	22.181977	2.969024	59.134611	21.927101	2.885216

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