An Approach to Determine the Weights for Junocam Ghost Images

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Abstract

Images taken by Junocam during Earth flyby show features suggesting the presence of structured ghost images. Assuming ghosts being weighted distortions of a known image, and assuming linear contributions of each ghost for a fixed pixel position, together with a non-singularity condition, the weights can be determined by means of linear algebra, and by related algebraic methods. [1]

[1]This document was typeset with LaTeX.

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1 Introduction

Images taken by Juno's Education and Outreach camera Junocam during the Earth flyby (EFB) in October 2013 are providing a first publicly available set of in-flight tests, similar to the images expected to be taken during Juno's Jupiter mission starting in mid-2016. So this article may be put in the context of [1, subsection 6.4], goal 3: "Provide data to the amateur image processing community and encourage them to produce a variety of products".

The images show features suggesting the presence of structured ghost images. This is in addition to the laboratory analysis of the stray light, where "there was little evidence of structure in the leakage" ([1, subsection 4.8]). Those ghost images may be a result of optical and/or electronic effects of the camera. They appear to be related to real world objects. Assuming a deterministic distortion of the real-world pinhole image for each type of ghost, the resulting actual transmitted images should be a combination of the ghosts and the primary image.

Junocam is CCD based. CCDs count photons. It's therefore plausible, that ghosts formed by reflected light contribute to the photon count in a linear way. The assumed ghosts are blurred and distorted images of a pinhole image, weighted differently for each pixel position. The weight is assumed to be constant over all images for a fixed pixel position. The actual weights are unknown. With the above assumptions the weights can be determined by means of linear algebra.

Raw Junocam images are composed of framelets of width 1648 pixels and height 128 pixels. Each of these framelets can be assigned to a color channel. There are four possible color channels, red, green, blue and CH_4.

To avoid motion blur due to the rotation of the Juno probe, Junocam supports a technique called Time Delay Integration (TDI). Very short shutter times don't require this mechanism. That's equivalent to TDI 1. For TDI 1, color channel and pixel position within a framelet determine uniquely the pixel position on the CCD chip. For simplicity, this article is restricted to TDI 1 mode.

Images are provided square root encoded. Linear data are obtained by squaring the raw color values.

Section 1 describes an approach to determine the weights for a single pixel of the ghosts. Section 2 generalizes this approach to framelets.

2 Determining the Weights of the Ghosts for a Pixel

Let $\mathbf{N}$ denote the set of the natural numbers, and $\mathbf{R}$ the field of the real numbers.

For each framelet j assume $n \in \mathbf{N}$ distorted and weighted images i being added to the resulting raw image. Consider n framelets of the same color channel. For a fixed pixel position on the CCD chip, and $1 \leq i, j \leq n$, let $x_{j,i} \in \mathbf{R}$ be the known brightness of the distorted image i in framelet j. Let $a_i \in \mathbf{R}$ be the weight for distorted image i at the fixed pixel position, independent of the framelet. Let $v_j \in \mathbf{R}$ be the linearized raw brightness of the fixed pixel in framelet j. Then for each framelet j, the linearized raw brightness of the fixed pixel is

$$v_j = x_{j,1} \cdot a_1 + \cdots + x_{j,n} \cdot a_n \ ,$$

or more formally

$$v_j = \sum_{i=1}^{n} x_{j,i} \cdot a_i \ .$$

This sum can be written as a scalar product of two vectors:

$$v_j = (x_{j,1}, \ldots x_{j,n}) \cdot \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} \ .$$

Consider all n framelets at once to get

$$\begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} = \begin{pmatrix} x_{1,1} & \cdots & x_{1,n} \\ \vdots & & \vdots \\ x_{n,1} & \cdots & x_{n,n} \end{pmatrix} \cdot \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} \ .$$

For the matrix

$$X := \begin{pmatrix} x_{1,1} & \cdots & x_{1,n} \\ \vdots & & \vdots \\ x_{n,1} & \cdots & x_{n,n} \end{pmatrix}$$

assumed to be regular, its inverse can be multiplied from the left to determine the weights a_i:

$$\begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} = \begin{pmatrix} x_{1,1} & \cdots & x_{1,n} \\ \vdots & & \vdots \\ x_{n,1} & \cdots & x_{n,n} \end{pmatrix}^{-1} \cdot \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} \ . \tag{1}$$

3 Generalization to Framelets

3.1 Set of Framelets with Commutative Ring Structure

Describe a framelet f of width $w \in \mathbf{N}$ and height $h \in \mathbf{N}$ as a vector $f = (f_1, \ldots, f_{w \cdot h})$, with $f_k \in \mathbf{R}$.

Definition 1 *The* sum $e + f$ *of two framelets* $e = (e_1, \ldots, e_{w \cdot h})$ *and* $f = (f_1, \ldots, f_{w \cdot h})$*, both of width* w *and height* h*, is defined by*

$$e + f := (e_1 + f_1, \ldots, e_{w \cdot h} + f_{w \cdot h}).$$

Definition 2 *The* additive inverse $-f$ *of a framelet* $f = (f_1, \ldots, f_{w \cdot h})$ *of width* w *and height* h*, is defined by*

$$-f := (-f_1, \ldots, -f_{w \cdot h}).$$

Definition 3 *The* product $e \cdot f$ *of two framelets* $e = (e_1, \ldots, e_{w \cdot h})$ *and* $f = (f_1, \ldots, f_{w \cdot h})$*, both of width* w *and height* h*, is defined by*

$$e \cdot f := (e_1 \cdot f_1, \ldots, e_{w \cdot h} \cdot f_{w \cdot h}).$$

Proposition 4 *The set of framelets of width* w *and height* h*, together with sum and product form a ring with* $0 := (0, \ldots, 0)$*, and* $1 := (1, \ldots, 1)$*.*

Additive neutral element: By definitrion of 0 and f, $0 + f = (0, \ldots, 0) + (f_1, \ldots, f_{w \cdot h})$. By defintion of the sum, that's equal to $(0 + f_1, \ldots, 0 + f_{w \cdot h})$. By the definition of the neutral element in the field **R**, that's $(f_1, \ldots, f_{w \cdot h}) = f$. Hence $0 + f = f$.

Similar component-wise verifications show $f + 0 = f$, $(d + e) + f = d + (e + f)$, $f + (-f) = 0$, and $e + f = f + e$. Hence the set of framelets of width w and height h, together with the sum and 0 form an abelian group, by definition.

The set of framelets of width w and height h, together with the product and 1 form a commutative monoid, since $1 \cdot f = f = f \cdot 1$ (neutral element), $e \cdot f = f \cdot e$ (comutativity), $d \cdot (e \cdot f) = (d \cdot e) \cdot f$ (associativity), again by component-wise verification.

The distributive law $d \cdot (e + f) = d \cdot e + d \cdot f$ is easily verified the same way.

All axioms of a commutative ring are verified. $\triangle$

(For the definition of "ring" see e.g. [2, subsection 2.1].)

Remark 5 *The set of framelets of width* w *and height* h*, together with sum and product do neither form a field nor a division ring with* $0 := (0, \ldots, 0)$*, and* $1 := (1, \ldots, 1)$*, in general.*

Counterexample: The non-zero framelet $(0, 1)$ of width 2 and height 1 has no multiplicative inverse. $\triangle$

3.2 Determining the Weights of the Ghosts for a Framelet

According to [2, subsection 2.3] the concept of invertible matrices can be generalized to matrices over a commutative ring.

Hence equation (1) can be applied to framelets instead of just single pixel positions.

A pixel-wise definition, however, covers more cases, since a single pixel singularity induces a singularity for the whole framlet.

To weaken the strict non-singularity condition for framelets, undefined pixel weights may be replaced by default values. Two types of undefined values may be considered, $0/0$, and $z/0$ with $z \neq 0$. This way equation (1) can be used for framelet matrices with singularities.

References

[1] C.J. Hansen, M.A. Caplinger, A. Ingersoll, M.A. Ravine, E. Jensen, S. Bolton, G. Orton. *Junocam: Juno's Outreach Camera.* Space Sci Rev DOI 10.1007/s11214-014-0079-x, Springer, 2014

[2] Nathan Jacobson. *Basic Algebra I, Second Edition.* W. H. Freeman and Company, New York, 1985.